

# Package ‘fgac’

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**Title** Generalized Archimedean Copula

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**Description** Bi-variate data fitting is done by two stochastic components: the marginal distributions and the dependency structure. The dependency structure is modeled through a copula. An algorithm was implemented considering seven families of copulas (Generalized Archimedean Copulas), the best fitting can be obtained looking all copula’s options (totally positive of order 2 and stochastically increasing models).

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cumulativemarg	<i>cumulativemarg</i>
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### Description

Auxiliary function that is used in copula fitting. This function works with different cumulative forms, like pnorm, pbeta, ... and transforms it as cumulative1 and / or cumulative2 in fitCBB function and OptimCBB function.

### Usage

```
cumulativemarg(cumulative, x, a)
```

**Arguments**

cumulative	can be pnorm, punif, pbeta, pempirical, ...
x	real vector
a	parameters associated with cumulative

**Value**

Cumulative distribution, evaluated in the vector x

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[match.arg](#), [pempirical](#)

**Examples**

```
#x<-rnorm(50,2,1)
#a<-cumulativemarg(pnorm,x,c(2,1))
#a1<-cumulativemarg(pempirical,x)
```

---

dirac1

*dirac1*

---

**Description**

Indicator function of the set A, where  $A = [u, \text{infinity})$

**Usage**

```
dirac1(u, x)
```

**Arguments**

u	real value
x	real value

**Value**

$\text{dirac1}(u,x)=1$  if  $x \geq u$  and  $\text{dirac1}(u,x)=0$  in other case.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[dirac2](#), [diracS1](#), [diracS2](#)

---

dirac2

*dirac2*

---

**Description**

Indicator function of the set  $A$ , where  $A = [u, \infty) \times [v, \infty)$

**Usage**

`dirac2(u, v, x, y)`

**Arguments**

<code>u</code>	real value
<code>v</code>	real value
<code>x</code>	real value
<code>y</code>	real value

**Value**

$\text{dirac2}(u, v, x, y) = 1$  if  $x \geq u$  and  $y \geq v$ , in other case  $\text{dirac2}(u, v, x, y) = 0$ .

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[dirac1](#), [diracS1](#), [diracS2](#)

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diracS1	<i>diracS1</i>
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**Description**

Indicator function of the set A, where  $A = (-\infty, u)$

**Usage**

`diracS1(u, x)`

**Arguments**

u	real value
x	real value

**Value**

`diracS1(u,x)=1` if  $x < u$  and `diracS1(u,x)=0` in other case.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[dirac1](#), [dirac2](#), [diracS2](#)

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diracS2	<i>diracS2</i>
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**Description**

Indicator function of the set A, where  $A = (-\infty, u) \times (-\infty, v)$ .

**Usage**

`diracS2(u, v, x, y)`

**Arguments**

u	real value
v	real value
x	real value
y	real value

**Value**

$\text{diracS2}(u,v,x,y)=1$  if  $x < u$  and  $y < v$ ;  $\text{diracS2}(u,v,x,y)=0$  in other case.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[dirac1](#), [dirac2](#), [diracS1](#)

---

fcopulamodel

*fcopulamodel*

---

**Description**

Auxiliary function that is used in copula fitting. This function works with different cumulative copulas, `fcopulamodel` transforms it as cumulatives in `fitCBB` function and `OptimCBB` function.

**Usage**

```
fcopulamodel(theta, delta, x, y, model = c("pCBB1", "pCBB2", "pCBB3", "pCBB4", "pCBB5",
      "pCBB6", "pCBB7", "pCMax", "pCMin"))
```

**Arguments**

<code>theta</code>	real parameter
<code>delta</code>	real parameter
<code>x</code>	real vector
<code>y</code>	real vector
<code>model</code>	bidimensional cumulative, can be any of the following : <code>pCBB1</code> , <code>pCBB2</code> , <code>pCBB3</code> , <code>pCBB4</code> , <code>pCBB5</code> , <code>pCBB6</code> , <code>pCBB7</code> , <code>pCMax</code> , <code>pCMin</code>

**Details**

If `model` is missing `fcopulamodel` works with `pCBB1`.

**Value**

Bidimensional cumulative. Specific form that can be used in copula fitting.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[match.arg](#), [pCBB1](#), [pCBB1](#), [pCBB2](#), [pCBB3](#), [pCBB4](#), [pCBB5](#), [pCBB6](#), [pCBB7](#), [pCMax](#), [pCMin](#)

**Examples**

```
#x<-runif(50)
#y<-runif(50)
#a<-fcopulamodel(2,3,x,y, model = "pCBB6")
```

---

FE1vector

*FE1vector*

---

**Description**

Empirical cumulative distribution

**Usage**

```
FE1vector(u, x)
```

**Arguments**

u	real vector
x	real vector (can be like u)

**Value**

empirical cumulative distribution from u sample, evaluated in the vector x.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[SOB2](#), [FE2](#)

**Examples**

```
# x<-rnorm(50)
# FE1vector(x,x)
# y<-rnorm(10)
# FE1vector(x,y)
```

FE2

*FE2***Description**

Bidimensional empirical cumulative distribution

**Usage**

FE2(u, v, x, y)

**Arguments**

u	real vector
v	real vector
x	real value (can be some component of u)
y	real value (can be some component of v)

**Details**

$$FE2(u, v, x, y) = \frac{1}{n} \sum_{i=1}^n I_{(u_i \leq x)} I_{(v_i \leq y)}, \quad u = (u_1, \dots, u_n), v = (v_1, \dots, v_n)$$

**Value**

Bidimensional empirical cumulative distribution from (u,v) sample, evaluated in the point (x,y)

**Author(s)**

Veronica Andrea Gonzalez-Lopez

**See Also**

[SOB2](#), [FE1vector](#)

**Examples**

```
#u<-matrix(c(1,3,5,1,6),nrow=5,ncol=1)
#FE2(u,u,6.5,3)
#u<-matrix(c(1:15),nrow=15,ncol=1)
#v<-matrix(c(16:30),nrow=15,ncol=1)
#FE2(u,v,5,35)
```

fitCBB

*fitCBB***Description**

Fitting an specific generalized archimedean copula

**Usage**

```
fitCBB(x, y, theta0, delta0, copulamodel = c("pCBB1", "pCBB2", "pCBB3", "pCBB4",
      "pCBB5", "pCBB6", "pCBB7", "pCMax", "pCMin"), m, step, deltamin, thetamin,
      test = c("wilcox.test", "t.test"), empcumulative = TRUE, cumulative1,
      cumulative2, parameters1, parameters2)
```

**Arguments**

x	real vector
y	real vector
theta0	parameter in the model pCBBi (in variable copulamodel). For default, theta0 is obtained from fitlambdas
delta0	parameter in the model pCBBi (in variable copulamodel). For default, delta0 is obtained from fitlambdas
copulamodel	specific model that we need to fit, it need to be one option from: pCBB1 (default), pCBB2, pCBB3, pCBB4, pCBB5, pCBB6, pCBB7, pCMax, pCMin
m	integer positive number (default=15)
step	real positive number (default=0.01)
deltamin	minimum value admitted for delta's domain (default=epsilon-see details)
thetamin	minimum value admitted for theta's domain (default=epsilon-see details)
test	test used for fitting selection, it need to be wilcox.test(default) or t.test
empcumulative	logical value, can be TRUE (default) or FALSE (see details)
cumulative1	marginal cumulative associated with x. Can be used pnorm, pbeta, pempirical,...(only used when empcumulative=FALSE)
cumulative2	marginal cumulative associated with y. Can be used pnorm, pbeta, pempirical,...(only used when empcumulative=FALSE)
parameters1	specifics parameters for cumulative1's definition
parameters2	specifics parameters for cumulative2's definition

## Details

The function constructs a neighbourhood around  $(\theta_0, \delta_0)$  for family specified in 'copulamodel', and using the test specified in 'test' the function search the best  $(\theta^*, \delta^*)$  in the neighbourhood such that  $\text{copulamodel}(\theta^*, \delta^*, u, v)$  is close to the bivariate empirical copula from  $(x, y)$ . Where  $(u, v) = (\text{cumulative1}(x), \text{cumulative2}(y))$ .  $m$  and  $\text{step}$  control the neighbourhood definition.  $\text{deltamin}$  and  $\text{thetamin}$  depend on the model worked. For default, we have, pCBB1:  $\text{deltamin}=1, \text{thetamin}=0.05$ ; pCBB2:  $\text{deltamin}=0.05, \text{thetamin}=0.05$ ; pCBB3:  $\text{deltamin}=1, \text{thetamin}=0.05$ ; pCBB4:  $\text{deltamin}=0.05, \text{thetamin}=0.05$ ; pCBB5:  $\text{deltamin}=0.05, \text{thetamin}=1$ ; pCBB6:  $\text{deltamin}=1, \text{thetamin}=1$ ; pCBB7:  $\text{deltamin}=0.05, \text{thetamin}=1$ . If  $\text{empcumulative}=\text{TRUE}$  like default, the algorithm uses for uniformization, empirical cumulative from  $x$  for  $x$  and empirical cumulative from  $y$  for  $y$ . If  $\text{empcumulative}=\text{FALSE}$ , we need to put an specific  $\text{cumulative1}$  and an specific  $\text{cumulative2}$ . If necessary,  $\text{parameters1}$  contains the special parameter(s) for  $\text{cumulative1}$  and  $\text{parameters2}$  contains the special parameter(s) for  $\text{cumulative2}$ .

## Value

Empirical	empirical copula from $(x, y)$
Copula	best copulamodel evaluated in $(u, v) = (\text{cumulative1}(x), \text{cumulative2}(y))$
fit	performance from the best copulamodel in the neighbourhood. Result: p.value in $\text{fit}[1]$ , delta in $\text{fit}[2]$ , theta in $\text{fit}[3]$
thetaj	theta's vector constructed in the neighbourhood
deltaj	delta's vector constructed in the neighbourhood
pthetaideltaj	p value matrix from each combination. The position $(i, j)$ represents the p value from 'test' in $\text{thetaj}(i), \text{deltaj}(j)$ for copulamodel.

## Author(s)

Veronica Andrea Gonzalez-Lopez

## References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

## See Also

[fitlambdas](#), [OptimCBB](#) ~~~

## Examples

```
#x<-rnorm(100)
#y<-x/10+rnorm(100)
#M<-fitCBB(x,y) # default fitting
#default: thetas0 and delta0 from fitlambdas function, m=15, step=0.01,
#copulamodel="pCBB1", test="wilcox.test", empcumulative=TRUE.
#
#M<-fitCBB(x,y,theta0=1.1,delta0=0.8,copulamodel="pCBB5",m=20,step=0.5,deltamin=0.1,thetamin=1.1,
```

```
#test="w",empcumulative=FALSE,cumulative1=pnorm,cumulative2=pnorm)
#
#x<-rnorm(100)
#y<-x/100+rnorm(100,5,2)
#M<-fitCBB(x,y,theta0=1.1,delta0=0.8,copulamodel="pCBB7",m=20,step=0.5,deltamin=0.1,thetamin=1.1,
#test="t",empcumulative=FALSE,cumulative1=pnorm,cumulative2=pnorm,parameters2=c(5,2))
```

fitlambdas

*fitlambdas*

## Description

The function tests the compatibility for each model pCBBi,  $i=1,2,3,4,5,6,7$ , pCMax and pCMin in relation to a proposal caudal measures: lambdaLE, lambdaUE. Also, this function gives theta and delta in function of lambdaLE and lambdaUE.

## Usage

```
fitlambdas(lambdaLE, lambdaUE)
```

## Arguments

lambdaLE	real number in [0,1]
lambdaUE	real number in [0,1]

## Details

NaN values can be used in lambdaLE and lambdaUE

## Value

For  $i=1,2,3,4,5,7$

BBi.model	is =TRUE BBi if the BBi model can be used and is = FALSE BBi in other case
BBi.theta	real value if BBi.model is =TRUE BBi and = NaN if BBi.model is = FALSE BBi
BBi.delta	real value if BBi.model is =TRUE BBi and = NaN if BBi.model is = FALSE BBi
BB6.model	is =TRUE BB6 if the BB6 model can be used and is =FALSE BB6 in other case
BB6.deltaxtheta	real value if BB6.model is =TRUE BB6 and =NaN if BB6.model is =FALSE BB6
CMin.model	is =TRUE CMin if the CMin model can be used and is =FALSE CMin in other case
CMax.model	is =TRUE CMax if the CMax model can be used and is =FALSE CMax in other case

**Author(s)**

Veronica Andrea Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB1](#), [pCBB2](#),[pCBB3](#),[pCBB4](#),[pCBB5](#),[pCBB6](#),[pCBB7](#)

**Examples**

```
#fitlambdas(0.3,0.7)
#fitlambdas(0.9,NaN)
#fitlambdas(0.18,0)
#fitlambdas(0.18,0)
#fitlambdas(0,0)
#fitlambdas(0,NaN)
#fitlambdas(1,1)
```

---

ftest

*ftest*

---

**Description**

Auxiliary function that is used in copula fitting. This function works with different two sample test, ftest transforms it as test in fitCBB function and OptimCBB function.

**Usage**

```
ftest(x, y, test = c("wilcox.test", "t.test"))
```

**Arguments**

x	real vector
y	real vector
test	can be wilcox.test or t.test

**Details**

form that work with two test, if test is missing test is defined for wilcox.test.

**Value**

test between x and y.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[match.arg](#), [wilcox.test](#), [t.test](#)

**Examples**

```
#x<-rnorm(100)
#y<-rnorm(100)
#ftest(x,y)
#ftest(x,y,test="t")
```

---

ivphiBB1

*ivphiBB1*


---

**Description**

Inverse Laplace's transform (phiBB1's inverse)

**Usage**

```
ivphiBB1(theta, delta, t)
```

**Arguments**

theta	positive, real parameter
delta	real parameter ( $\geq 1$ )
t	real vector

**Value**

return the value for the inverse in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**[phiBB1](#)**Examples**

```
#a<-phiBB1(0.5,1.5,c(1,6))
#b<-ivphiBB1(0.5,1.5,c(a[5],a[6]))
```

---

*ivphiBB2**ivphiBB2*

---

**Description**

Inverse Laplace's transform ([phiBB2](#)'s inverse)

**Usage**

```
ivphiBB2(theta, delta, t)
```

**Arguments**

theta	positive, real parameter
delta	positive, real parameter
t	real vector

**Value**

return the value for the inverse in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03; Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**[phiBB2](#)

**Examples**

```
#a<-phiBB2(0.3,2.6,c(2,3,4))
#b<-ivphiBB2(0.3,2.6,c(a[6],a[7],a[8]))
```

---

ivphiBB3

*ivphiBB3*


---

**Description**

Inverse Laplace's transform (phiBB3's inverse)

**Usage**

```
ivphiBB3(theta, delta, t)
```

**Arguments**

theta	positive, real parameter
delta	real parameter ( $\geq 1$ )
t	real vector

**Value**

return the value for the inverse in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[phiBB3](#)

**Examples**

```
#a<-phiBB3(0.2,4,c(0.2,0.3,0.4))
#b<-ivphiBB3(0.2,4,c(a[6],a[7],a[8]))
```

---

`ivphiBB6`*ivphiBB6*

---

**Description**

Inverse Laplace's transform (phiBB6's inverse)

**Usage**

```
ivphiBB6(theta, delta, t)
```

**Arguments**

theta	real parameter ( $\geq 1$ )
delta	real parameter ( $\geq 1$ )
t	real vector

**Value**

return the value for the inverse in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[phiBB6](#)

**Examples**

```
#a<-phiBB6(1.1,2.1,c(0.55,0.66,0.77,0.88))  
# b<-ivphiBB6(1.1,2.1,c(a[7],a[8],a[9],a[10]))
```

---

`ivphiBB7`*ivphiBB7*

---

**Description**

Inverse Laplace's transform (phiBB7's inverse)

**Usage**

```
ivphiBB7(theta, delta, t)
```

**Arguments**

theta	real parameter ( $\geq 1$ )
delta	positive, real parameter
t	real vector

**Value**

return the value for the inverse in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[phiBB7](#)

**Examples**

```
#a<-phiBB7(1.1,0.8,c(0.55,0.66,0.77,0.88))  
#b<-ivphiBB7(1.1,0.8,c(a[7],a[8],a[9],a[10]))
```

---

`ivpsiGumbel`*ivpsiGumbel*

---

**Description**

Inverse Laplace's transform (psiGumbel's inverse)

**Usage**

```
ivpsiGumbel(delta, t)
```

**Arguments**

delta            real ( $\geq 1$ ), parameter

t                real positive vector

**Value**

return the value for the inverse transform in the vector t

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[psiGumbel](#)

**Examples**

```
#delta=2, vector=c(1,2,3,4)
#ivpsiGumbel(2,c(1,2,3,4))
```

---

`ivpsiKS`*ivpsiKS*

---

**Description**

Inverse Laplace's transform (psiKS's inverse)

**Usage**

```
ivpsiKS(delta, t)
```

**Arguments**

<code>delta</code>	real and positive parameter
<code>t</code>	real positive vector

**Value**

return the value for the inverse transform in the vector `t`

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[psiKS](#)

**Examples**

```
#a<-psiKS(0.4,c(1,1.5,2,2.5,3,3.5))
#b<-ivpsiKS(0.4,c(a[8],a[9],a[10],a[11],a[12],a[13]))
#
```

---

KGalambos

*KGalambos*

---

**Description**

Galambos's cumulative. Stochastically increasing copula.

**Usage**

```
KGalambos(u, v, delta)
```

**Arguments**

u	real in [0,1]
v	real in [0,1]
delta	real and positive parameter

**Value**

Cumulative value for (u,v) obtained using Galambos's cumulative

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB4](#), [pCBB5](#), [psiKS](#), [psiGumbel](#)

**Examples**

```
#u=0.6, v=0.7, delta=7  
#KGalambos(0.6, 0.5, 7)
```

OptimCBB

*OptimCBB***Description**

The best fitting into the generalized archimedean copula class is selected

**Usage**

```
OptimCBB(x, y, m, step, test = c("wilcox.test", "t.test"), empcumulative = TRUE,
cumulative1, cumulative2, parameters1, parameters2)
```

**Arguments**

x	real vector
y	real vector
m	integer positive number (default=15)
step	real positive number (default=0.01)
test	test used for fitting selection, it have to be wilcox.test (default) or t.test
empcumulative	logical value, can be TRUE (default) or FALSE (see details)
cumulative1	marginal cumulative associated with x. Can be used pnorm, pbeta, pempirical,...(only used when empcumulative=FALSE)
cumulative2	marginal cumulative associated with y. Can be used pnorm, pbeta, pempirical,...(only used when empcumulative=FALSE)
parameters1	specifics parameters for cumulative1's definition
parameters2	specifics parameters for cumulative2's definition

**Details**

The function cheks the compatibility of each family using 'fitlambdas' then, the function 'fitCBB' is applied for each possible family . Partial and global good fit are showed.

**Value**

Empirical	empirical copula for (x,y)
Copula	best copulamodel evaluated in (u,v)=cumulative1(x),cumulative2(y)
OptimumFit	performance from the best copulamodel in the neighbourhood and between all copula's families pCBB1,..., pCBB7, pCMax, pCMin. Family in OptimumFit[1]; p.value in OptimumFit[2], delta in OptimumFit[3], theta in OptimumFit[4], MSE in OptimumFit[5]
Initial.BBi	For i in 1,...,7. Initial values for BBi family provided by the fitlambdas function. If Initial.BBi[1]=FALSE BBi, the BBi family is excluded (because empirical evidence from the data shows that this family is not appropriated). If Initial.BBi[1]=TRUE BBi, theta e delta suggested from fitlambdas function is showed in Initial.BBi[2] and Initial.BBi[3] respectively.

`Final.BBi` For  $i$  in 1,...,7, we have the characteristics from the best fit in  $BBi$  family. If `Final.BBi[1]=FALSE`  $BBi$ , the  $BBi$  family is excluded (only when the family was excluded in `Initial. BBi`). In other case ,`Final.BBi[1]=p.value` (from test); the best  $\theta$  e  $\delta$  are showed in `Final.BBi[2]` and `Final.BBi[3]` respectively.

`Initial.CMax` (`Initial.CMin`)  
like `Initial.BBi` (in this kind of component  $\theta$  and  $\delta$  do not have sense)

`Final.CMax` (`Final.CMin`)  
like `Final.BBi`

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. ‘Bi-variate Data Modeling Through Generalized Archimedean Copula’ RT-MAE 2003-03

**See Also**

[fitCBB](#), [fitlambdas](#)

**Examples**

```
#x<-rnorm(100)
#y<-x/100+rnorm(100,5,2)
#M<-OptimCBB(x=x,y=y)
#
#x<-rbeta(50,2,3)
#y<-0.5*x+rgamma(50,1,2)
#M<-OptimCBB(x,y,m=30,step=0.5,test="t",empcumulative=TRUE)
#M<-OptimCBB(x,y,m=30,step=0.5,test="w",empcumulative=FALSE,cumulative1=pbeta,
#cumulative2=pempirical,parameters1=c(2,3))
```

---

`pCBB1`

*pCBB1*

---

**Description**

Cumulative generalized archimedean Copula  $BB1$

**Usage**

`pCBB1(theta, delta, s, t)`

**Arguments**

theta	real and positive parameter
delta	real parameter ( $\geq 1$ )
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula1](#), [phiBB1](#), [psiGumbel](#)

**Examples**

```
a<-pCBB1(2,3,matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5),matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5))
```

---

pCBB2

*pCBB2*

---

**Description**

Cumulative generalized archimedean Copula BB2

**Usage**

```
pCBB2(theta, delta, s, t)
```

**Arguments**

theta	real and positive parameter
delta	real and positive parameter
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula1](#), [phiBB2](#), [psiKS](#)

**Examples**

```
#a<-pCBB2(0.9,0.3,matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5),matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5))
```

---

pCBB3

*pCBB3*

---

**Description**

Cumulative generalized archimedean Copula BB3

**Usage**

```
pCBB3(theta, delta, s, t)
```

**Arguments**

theta	real and positive parameter
delta	real parameter ( $\geq 1$ )
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula1](#), [phiBB3](#), [psiKS](#)

**Examples**

```
#a<-pCBB3(0.2,1.7,matrix(c(0.11,0.22,0.34,0.21,0.35),nrow=5),
#matrix(c(0.55,0.77,0.65,0.79,0.76),nrow=5))
```

---

pCBB4

*pCBB4*

---

**Description**

Cumulative generalized archimedean Copula BB4

**Usage**

```
pCBB4(theta, delta, s, t)
```

**Arguments**

theta	real and positive parameter
delta	real and positive parameter
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula2](#), [psiKS](#), [KGalambos](#)

**Examples**

```
#s<-matrix(c(0.1,0.2,0.3,0.4,0.5),nrow=5)
#t<-matrix(c(0.15,0.28,0.31,0.49,0.51),nrow=5)
#a<-pCBB4(0.5,0.9,s,t)
```

---

pCBB5

*pCBB5*

---

**Description**

Cumulative generalized archimedean Copula BB5

**Usage**

```
pCBB5(theta, delta, s, t)
```

**Arguments**

theta	real parameter( $\geq 1$ )
delta	real and positive parameter
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. ‘Bi-variate Data Modeling Through Generalized Archimedean Copula’ RT-MAE 2003-03. Harry Joe. ‘Multivariate Models and Dependence Concepts’ Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula2](#), [psiGumbel](#), [KGalambos](#)

**Examples**

```
#s<-matrix(c(0.1,0.2,0.3,0.4,0.5),nrow=5)
#t<-matrix(c(0.15,0.28,0.31,0.49,0.51),nrow=5)
#a<-pCBB5(1.5,0.9,s,t)
```

pCBB6

*pCBB6***Description**

Cumulative generalized archimedean Copula BB6

**Usage**

```
pCBB6(theta, delta, s, t)
```

**Arguments**

theta	real parameter ( $\geq 1$ )
delta	real parameter ( $\geq 1$ )
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pcopula1](#), [phiBB6](#), [psiGumbel](#)

**Examples**

```
#a<-pCBB6(3,1.7,matrix(c(0.11,0.22,0.34,0.21,0.35),nrow=5),
#matrix(c(0.55,0.77,0.65,0.79,0.76),nrow=5))
```

---

pCBB7

*pCBB7*

---

### Description

Cumulative generalized archimedean Copula BB7

### Usage

```
pCBB7(theta, delta, s, t)
```

### Arguments

theta	real parameter ( $\geq 1$ )
delta	real and positive parameter
s	real vector
t	real vector

### Value

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters.

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

### See Also

[pcopula1](#), [phiBB7](#), [psiKS](#)

### Examples

```
#s<-matrix(c(0.1,0.2,0.3,0.4,0.5),nrow=5)
#t<-matrix(c(0.15,0.28,0.31,0.49,0.51),nrow=5)
#a<-pCBB7(2,0.9,s,t)
```

---

pCMax

*pCMax*

---

### Description

Cumulative copula Frechet's bound, pCMax

### Usage

```
pCMax(theta, delta, s, t)
```

### Arguments

theta	is missing
delta	is missing
s	real vector
t	real vector

### Value

returns the values from bidimensional cumulative for (s,t) sample.

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

### See Also

[pCMin](#)

### Examples

```
#a<-pCMax(s=matrix(c(0.9,0.2,0.4,0.5),nrow=4),t=matrix(c(0.2,0.33,0.5,0.2),nrow=4))
```

pCMin

*pCMin*

---

**Description**

Cumulative copula Frechet's bound, pCMin

**Usage**

```
pCMin(theta, delta, s, t)
```

**Arguments**

theta	is missing
delta	is missing
s	real vector
t	real vector

**Value**

returns the values from bidimensional cumulative for (s,t) sample.

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCMax](#)

**Examples**

```
#x<-rnorm(50,0,1)
#y<-1-x+rnorm(50,0.05,0.1)
#plot(x,y)
#a<-pCMin(s=x,t=y)
```

---

pcopula1                      *pcopula1*

---

### Description

Generator of generalized archimedean copula. Different kind of cumulative copulas can be obtained using pcopula1, for example pCBBi, i=1,2,3,6,7.

### Usage

```
pcopula1(theta, delta, psi, phi, ivpsi, ivphi, s, t)
```

### Arguments

theta	parameter, real and positive value
delta	parameter, real and positive value
psi	Laplace transformation
phi	Laplace transformation
ivpsi	psi's inverse
ivphi	phi's inverse
s	real vector
t	real vector

### Value

returns the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters and Laplace transformation.

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03

### See Also

[pCBB1](#), [pCBB2](#), [pCBB3](#), [pCBB6](#),[pCBB7](#)

### Examples

```
#pcopula1(2,3,psiGumbel,phiBB6,ivpsiGumbel,ivphiBB6,matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5),
#matrix(c(0.9,0.2,0.4,0.1,0.3),nrow=5))
```

---

 pcopula2

*pcopula2*


---

### Description

Generator of generalized archimedean copula. Different cumulative copulas can be obtained using pcopula2, for example pCBBi, i=4,5.

### Usage

```
pcopula2(theta, delta, psi, v1, ivpsi, v2, s, t)
```

### Arguments

theta	parameter, real and positive value
delta	parameter, real and positive value
psi	Laplace transformation
v1	real number
ivpsi	psi's inverse
v2	real number
s	real vector
t	real vector

### Value

return the values from bidimensional cumulative for (s,t) sample using (theta,delta) parameters and Laplace transformation.

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03

### See Also

[pCBB4](#), [pCBB5](#)

### Examples

```
#pcopula2(2.5,3,psiGumbel,1,ivpsiGumbel,1,
#matrix(c(0.9,0.7,0.2,0.5,0.4),nrow=5),matrix(c(0.9,0.7,0.2,0.5,0.4),nrow=5))
```

---

pempirical	<i>pempirical</i>
------------	-------------------

---

**Description**

Empirical cumulative distribution

**Usage**

```
pempirical(x, arg)
```

**Arguments**

x	real vector
arg	real vector (can be like x)

**Details**

pempirical can be used like pnorm, punif, pbeta,...

**Value**

empirical cumulative distribution for x sample, evaluated in the vector arg. If arg is missing, arg<-x.

**Author(s)**

Veronica A. Gonzalez-Lopez

**See Also**

[cumulativemarg](#), [pnorm](#)

**Examples**

```
#x<-rnorm(50,2,1)
#pempirical(x)
```

---

`phiBB1`*phiBB1*

---

**Description**

Laplace's transform. This function is associated with BB1 Copula

**Usage**

```
phiBB1(theta, delta, s)
```

**Arguments**

theta	positive, real parameter
delta	real parameter ( $\geq 1$ )
s	real vector

**Value**

return the value for the transform in the vector s

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB1](#), [psiKS](#)

**Examples**

```
#phiBB1(0.5, 1.5, c(1, 6))
```

---

`phiBB2`*phiBB2*

---

**Description**

Laplace's transform. This function is associated with BB2 Copula

**Usage**

```
phiBB2(theta, delta, s)
```

**Arguments**

theta	positive, real parameter
delta	positive, real parameter
s	real vector

**Value**

return the value for the transform in the vector s

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB2](#), [psiKS](#)

**Examples**

```
#theta=0.3,delta=2.6, s=c(2,3,4)
#phiBB2(0.3,2.6,c(2,3,4))
```

---

phiBB3

*phiBB3*

---

### Description

Laplace's transform. This function is associated with BB3 Copula

### Usage

```
phiBB3(theta, delta, s)
```

### Arguments

theta	positive, real parameter
delta	real parameter ( $\geq 1$ )
s	real vector

### Value

return the value for the transform in the vector s

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

### See Also

[pCBB3](#), [psiKS](#)

### Examples

```
#theta=0.2,delta=4, s=c(0.2,0.3,0.4)
#phiBB3(0.2,4,c(0.2,0.3,0.4))
```

---

`phiBB6`*phiBB6*

---

**Description**

Laplace's transform. This function is associated with BB6 Copula

**Usage**

```
phiBB6(theta, delta, s)
```

**Arguments**

theta	real parameter ( $\geq 1$ )
delta	real parameter ( $\geq 1$ )
s	real vector

**Value**

return the value for the transform in the vector s

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB6](#), [psiGumbel](#)

**Examples**

```
#theta=1.1,delta=2.1,s=c(0.55,0.66,0.77,0.88)
#phiBB6(1.1,2.1,c(0.55,0.66,0.77,0.88))
```

---

phiBB7

*phiBB7*

---

### Description

Laplace's transform. This function is associated with BB7 Copula

### Usage

```
phiBB7(theta, delta, s)
```

### Arguments

theta	real parameter ( $\geq 1$ )
delta	positive, real parameter
s	real vector

### Value

return the value for the transform in the vector s

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03; Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

### See Also

[pCBB7](#), [psiKS](#)

### Examples

```
#theta=1.1,delta=0.8,s=c(0.55,0.66,0.77,0.88)
#phiBB7(1.1,0.8,c(0.55,0.66,0.77,0.88))
```

---

`psiGumbel`*psiGumbel*

---

**Description**

Laplace's transform. This function is associated with Gumbel Archimedean Copula

**Usage**

```
psiGumbel(delta, s)
```

**Arguments**

`delta` parameter, real ( $\geq 1$ )

`s` real positive vector

**Value**

return the value for the transform in the vector `s`

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[pCBB1](#), [pCBB5](#), [pCBB6](#)

**Examples**

```
#Gumbel' TL with delta=1.7 and s=c(1:6)
#psiGumbel(1.7, c(1:6))
```

---

psiKS

*psiKS*

---

### Description

Laplace's transform. This function is associated with Kimeldorf-Sampson Archimedean Copula

### Usage

```
psiKS(delta, s)
```

### Arguments

delta	real and positive parameter
s	real positive vector

### Value

return the value for the transform in the vector s

### Author(s)

Veronica A. Gonzalez-Lopez

### References

Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

### See Also

[pCBB2](#), [pCBB3](#), [pCBB4](#), [pCBB7](#)

### Examples

```
#delta=0.4, s=c(1,1.5,2,2.5,3,3.5)
#psiKS(0.4,c(1,1.5,2,2.5,3,3.5))
```

SOB2

SOB2

**Description**

Bidimensional empirical survival function

**Usage**

SOB2(u, v, x, y)

**Arguments**

u	real vector
v	real vector
x	real vector
y	real vector

**Details**

$$SOB2(u, v, x, y) = \frac{1}{n} \sum_{i=1}^n I_{(u_i > x)} I_{(v_i > y)}, \quad u = (u_1, \dots, u_n), \quad v = (v_1, \dots, v_n)$$

**Value**

Bidimensional empirical survival function for vector (u,v), evaluated in (x,y)

**Author(s)**

Veronica A. Gonzalez-Lopez

**References**

Veronica A. Gonzalez-Lopez and Nelson I. Tanaka. 'Bi-variate Data Modeling Through Generalized Archimedean Copula' RT-MAE 2003-03. Harry Joe. 'Multivariate Models and Dependence Concepts' Monogra. Stat. & Appl. Probab. 73. Chapman and Hall (1997)

**See Also**

[FE1vector](#), [FE2](#)

**Examples**

```
#u<-matrix(c(1,3,5,1,6),nrow=5)
#SOB2(u,u,6.5,3)
#u<-matrix(c(1:15),nrow=15)
#v<-matrix(c(16:30),nrow=15)
#SOB2(u,v,10,24)
```

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