Supplement for
Information aggregation with costly reporting

Martin J. Osborne, Jeffrey S. Rosenthal, and Colin Stewart

December 21, 2018

1. Two agents

When there are two or more agents, the structure of the equilibria is complex, and we do not have a detailed characterization for general signal distributions.

To illustrate the complexity, suppose that the signal distributions are triangular ($f_0(x) = 2(1-x)$ and $f_1(x) = 2x$) and $q = \frac{1}{2}$. Equilibria in which the moderator chooses option 0 if no signal is revealed take the following forms. For $c < \frac{3}{2} - \sqrt{2}$, the game has an equilibrium in which each agent reveals signals outside $[\frac{1}{2} - (1 + \frac{1}{2}\sqrt{2})\sqrt{c}, \frac{1}{2} + \frac{1}{2}\sqrt{2}\sqrt{c}]$ and does not reveal signals in the interior of this interval, and for $c > \frac{3}{2} - \sqrt{2}$ it has an equilibrium in which each agent reveals signals greater than $\frac{1}{3}(1 + \sqrt{1+3c})$ and does not reveal smaller signals (the blue equilibrium in Figure 1). In this equilibrium, whenever exactly one agent reveals her signal, the moderator chooses the option favored by that signal. If both agents reveal signals, the moderator chooses the option favored by the stronger signal. At the $\overline{x}$ threshold the incentive for an agent to reveal her signal is driven by the possibility that the other agent may not reveal her signal, whereas at the $\underline{x}$ threshold this incentive is driven by the possibility that the other agent will reveal a signal that is close to the threshold but less than $1 - \underline{x}$.

In addition, for $c \le \frac{1}{16}$ for each $\underline{x} \in [0, \frac{1}{2} - 2\sqrt{c}]$ the game has an equilibrium in which each agent reveals signals outside $[\underline{x}, \underline{x} + 2\sqrt{c}]$ and does not reveal signals in the interior of this interval.[1] Note that in each of these equilibria an agent reveals the signal $\frac{1}{2}$ even

[1] This continuum of equilibria collapses to the equilibria with $\underline{x} \approx 0$ and $\underline{x} \approx \frac{1}{2}\sqrt{c}$ for q slightly different

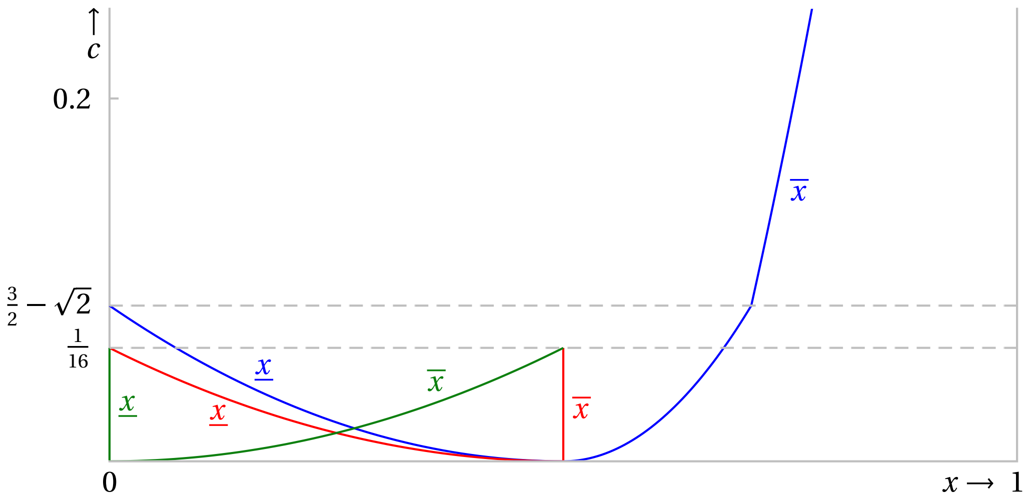

Figure 1. The equilibria when there are two individuals and the moderator chooses option 0 when no signal is revealed, for triangular signal distributions and $q = \frac{1}{2}$. For each value of $c \leq \frac{1}{16}$ there is a continuum of equilibria between the green and red equilibria, with $\overline{x} - \underline{x} = 2\sqrt{c}$.

though it is completely uninformative about the state. She does so because if she did not then the moderator would believe that her signal favored state 0 and would choose option 0 even if the other agent revealed a signal moderately in favor of state 1 (although option 1 is best given these signals).

For each of these equilibria there is an equilibrium in which the roles of options 0 and 1 are reversed, and the moderator chooses option 1 if no signal is revealed.

For the same triangular signal distributions and values of q other than $\frac{1}{2}$ analytic expressions for the equilibrium thresholds seem not to exist. Numerical computations for specific values of c reveal multiple isolated equilibria that depend on the value of q nonmonotonically in ways that are difficult to interpret.

2. General payoffs

We argue that our assumption that the payoffs from the options are 0 and 1 is essentially without loss of generality. First, note that the payoff of 0 for an option that differs from the state is simply a normalization; if the payoff to option $1-s$ in state s is $b \neq 0$, we can subtract b from the payoff to each action in state s. Now, taking the payoff to option $1-s$ in state s to be 0, denote by b_s the payoff from option s in state s. If b_0 and b_1 are

from $\frac{1}{2}$.

both negative, we can relabel the states to make them positive. If one is negative while the other is positive, the moderator has a dominant action and the game is trivial. Accordingly, suppose both are positive. Let a denote the option chosen by the moderator. Given any strategy profile, the expected payoff of any type x_i of agent i is

$$\begin{aligned}
&qb_0 \Pr(a=0 \mid s=0, x_i) + (1-q)b_1 \Pr(a=1 \mid s=1, x_i) - c\sigma_i(x_i) \\
&\quad = (qb_0 + (1-q)b_1)\left[q' \Pr(a=0 \mid s=0, x_i) + (1-q')\Pr(a=1 \mid s=1, x_i) - c'\sigma_i(x_i)\right],
\end{aligned}$$

where

$$q' = \frac{qb_0}{qb_0 + (1-q)b_1} \quad \text{and} \quad c' = \frac{c}{qb_0 + (1-q)b_1}.$$

Hence the payoffs for type x_i are proportional to those in the game with prior q', cost c', and payoff 1 from action s in each state s. The same transformation applies to the moderator. Therefore, a strategy profile is an equilibrium of the original game if and only if it is an equilibrium of this transformed game.