

# Group-based Criminal Trajectory Analysis using Cross-Validation Criteria

J.D. Nielsen<sup>1</sup>, J.S. Rosenthal<sup>2</sup>, Y. Sun<sup>3</sup>,  
D.M. Day<sup>4</sup>, I. Bevc<sup>5</sup>, and T. Duchesne<sup>6</sup>

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<sup>1</sup> School of Mathematics and Statistics, Carleton University, Ottawa, Ontario, Canada, K1S 5B6.

<sup>2</sup> Department of Statistics, University of Toronto, 100 St. George Street, Toronto, Ontario, Canada, M5S 3G3.

<sup>3</sup> Analytical Genetics Technology Centre, Princess Margaret Hospital, University Health Network, 620 University Avenue, Toronto, Ontario, Canada, M5G 2L7.

<sup>4</sup> Department of Psychology, Ryerson University, 350 Victoria Street, Toronto, Ontario, Canada, M5B 2K3.

<sup>5</sup> The Hincks-Dellcrest Centre, 1645 Sheppard Avenue West, Toronto, Ontario, Canada, M3M 2X4.

<sup>6</sup> Département de mathématiques et de statistique, Pavillon Alexandre-Vachon, Université Laval, 1045, av. de la Médecine, Québec City, Québec, Canada, G1V 0A6.

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## Abstract

In this paper, we discuss the challenge of determining the number of classes in latent class models for criminal trajectories. We argue that the traditional method of using either the *Proc Traj* or *Mplus* package to compute and maximize the Bayesian Information Criterion (BIC) is fundamentally flawed: *Proc Traj* and *Mplus* do not always compute BIC accurately, and furthermore BIC does not always indicate an appropriate number of groups even when computed correctly. As an alternative, we propose the new software package *crimCV* and the methodology of *cross-validation error* (CVE) to determine the number of classes in a fair and reasonable way. In the present paper, we apply the new methodology to two samples of  $N = 378$  and  $N = 386$  male juvenile offenders whose criminal behavior was tracked from late childhood/early adolescence into adulthood. We show that *crimCV* and CVE provide a superior method for determining the number of latent classes in these cases. These results suggest that cross-validation may be the best available method of determining the number of latent classes  $K$  for group-based trajectory analysis.

**Key Words and Phrases:** Group-based trajectory analysis; juvenile offenders; zero-inflated-poisson (ZIP); cross-validation; Bayesian information criterion; *crimCV*.

# 1 Introduction

Group-based trajectory models are a valuable method of modeling the relationship between age and criminal behavior in an effort to uncover the underlying or latent heterogeneity of the sample. It is widely known that criminal offenders are a diverse and varied population. These techniques are able to cluster individuals into small numbers of groups that show statistically similar trajectories in terms of rate or severity of offending over time. One challenge to these methods is to determine the number of groups to use in such a model. The most common approach is to optimize the Bayesian Information Criterion (BIC) (e.g. D'Unger, Land, McCall, & Nagin, 1998; Kass & Raftery, 1995; Raftery, 1995), using the software *Proc Traj* (Jones, 2001; Jones, Nagin, & Roeder, 2001; Jones, & Nagin, 2007). It is already known that such optimization is problematic (e.g. Nagin, 2005; Kreuter & Muthen, 2008).

In this paper we argue that this approach is fundamentally flawed in that *Proc Traj* and *Mplus* often fail to accurately compute and optimize BIC and, furthermore, BIC does not always indicate an appropriate number of groups even when computed correctly. As an alternative, we propose the new software package *crimCV* and the methodology of *cross-validation error* (CVE) to determine the number of classes in latent class models.

## 1.1 Previously-Known Problems with BIC

The BIC (Schwartz, 1978) is very commonly used in analyses such as those described in Section 2.1 of this paper. However, it is known to be problematic and often grossly overestimates the appropriate number of classes (Ward, Day, Bevc, Sun, Rosenthal, & Duchesne, 2010). For example, the book by Nagin (2005, pp. 74–75), *Group-Based Modeling of Development*, contains an entire section entitled “When BIC Is Not Useful in Identifying the Best Model,” which notes that:

BIC does not always cleanly identify a preferred number of groups. Instead,

in some applications the BIC score continues to increase as more groups are added. In such instances, more subjective criteria based on domain knowledge and the objectives of the analysis must be used to select the number of groups to include in the model. . . . [In a particular study] BIC continued to improve for this measurement series as more groups were added.

Similarly, Loughran and Nagin (2006, p. 259; see also Blokland, Nagin, & Nieuwbeerta, 2005) conducted an application and observed that:

In this application, the BIC was not helpful in identifying a preferred model because over the range of models explored, BIC monotonically increased with the number of groups. We settled on the four-group model . . . for several [unrelated] reasons.

In D’Unger, Land, and McCall (2002), the BIC again increases monotonically with the number of classes and they are forced to cut off this number when they can no longer invert a certain Hessian matrix as required to assess their standard errors. Similarly, in Piquero, Blumstein, Brame, Haapanen, Mulvey, and Nagin (2001), BIC again calls for more and more groups, until with  $K = 7$  their models “failed to converge on a solution”, forcing them to “settle on six-class models”. Finally, Yessine and Bonta (2009, pp. 446–447), using *MPlus* rather than the *Proc Traj*, also found that BIC is monotonically increasing with group number, but settled on using two groups after determining that “the mixture models failed to converge to a trustworthy solution when more than two groups were specified”. Such decisions are *ad hoc*, forced by computational issues rather than genuinely indicating optimality of choice of number of groups, and are thus quite unsatisfactory since the number of groups is selected by software limitations rather than by optimally modeling the data. Thus, despite the widespread use of BIC for determining the number of latent classes, this approach is well-known to be problematic, creating difficulties for researchers in this field. As noted by Nylund, Asparouhov and Muthén (2007, p. 537):

To date, there is not common acceptance of the best criteria for determining the number of classes in mixture modeling, despite various suggestions. This is a critical issue in the application of these models, because classes are used for interpreting results and making inferences.

Likewise, Eggleston, Laub and Sampson (2004; see also Nagin, 1999) write:

Although the Bayesian Information Criterion has been emphasized as the primary criterion to assess the optimal number of groups, the model selection process is often more complex and thus, group selection remains somewhat subjective.

## 1.2 Goal of this Paper

In this paper, we take these concerns about BIC two steps further. Firstly, we argue that the standard software packages *Proc Traj* and *Mplus* do not always compute BIC accurately, potentially leading to incorrect conclusions. This leads us to develop our own software package, *crimCV*. Secondly, we argue that even when computed correctly, BIC is a flawed criterion that often argues for too many groups without justification. This leads us to propose the use of CVE as alternative and often superior criterion.

## 2 The Models and Data

In this section we provide background information about the model and data and approach that we will consider.

### 2.1 The ZIP Latent Class Model

We first summarize the standard zero-inflation Poisson (ZIP) statistical model (Lambert, 2002) that we shall use to model criminal trajectories (Jones et al., 2001; Nagin, 2005).

Let  $Y_{ij}$  be the number of offenses by individual  $i$  at age  $j$  (in years), for  $1 \leq i \leq N$  and  $L \leq j \leq U$ . (Here  $L$  is the lower bound on offense ages, while  $U$  is the upper bound; e.g. for our first data set  $L = 8$  and  $U = 38$ , while for our second data set  $L = 9$  and  $U = 38$ .) To specify the statistical model, we need to specify the probability that  $Y_{ij} = y_{ij}$  for all  $i$  and  $j$ . For shorthand, let  $Y_i = (Y_{i,L}, Y_{i,L+1}, \dots, Y_{i,U})'$ , and  $y_i = (y_{i,L}, y_{i,L+1}, \dots, y_{i,U})'$ .

We assume that each individual  $i$  can be in one of  $K$  different latent classes, so that

$$P(Y_i = y_i) = \sum_{k=1}^K p_k P(Y_i = y_i \mid \text{individual } i \text{ is in class } k), \quad (1)$$

for some unknown probabilities  $\{p_k\}$  with  $0 < p_k < 1$  and  $\sum_{k=1}^K p_k = 1$ . We further make the standard assumption (which surely is not strictly true, but which greatly simplifies the mathematics) that, conditional on individual  $i$  being in class  $k$ , the number of offenses  $Y_{ij}$  are independent for different ages  $j$  so that equation (1) becomes

$$P(Y_i = y_i) = \sum_{k=1}^K p_k \prod_{j=L}^U P(Y_{ij} = y_{ij} \mid \text{individual } i \text{ is in class } k). \quad (2)$$

Equation (2) requires us to specify the probabilities for the  $Y_{ij}$  conditional on being in class  $k$ . To do this, we follow the zero-inflation Poisson (ZIP) model, which assumes that if individual  $i$  belongs to class  $k$ , then the distribution of  $Y_{ij}$  is a mixture of a Poisson distribution together with excess probability of zero offenses (corresponding to being in a “non-criminal” state). If we wish, we can also include year-by-year exposure times (i.e., times-at-risk),  $t_{ij}$ , which indicate the fraction of the year  $j$  in which individual  $i$  was not in secure custody. (Thus, the  $t_{ij}$  are all between 0 and 1, and can be taken to be 1 if time-at-risk adjustments are not available or required. That is,  $0 \leq t_{ij} \leq 1$ , with  $t_{ij} = 1$  indicating that individual  $i$  was at large for the entire year  $j$ .) Hence, the conditional probability distribution for  $Y_{ij}$  conditional on individual  $i$  being in class  $k$  becomes:

$$(Y_{ij} \mid \text{individual } i \text{ is in class } k) \sim (1 - q_j^k) \text{Poisson}(t_{ij}\lambda_j^k) + q_j^k \delta_0 \quad (3)$$

for some (unknown) mixture probabilities  $q_j^k$  and intensities  $\lambda_j^k$ , where  $\delta_0$  is a point-mass at zero. That is,

$$\begin{aligned} P(Y_{ij} = y_{ij} \mid \text{individual } i \text{ is in class } k) &= (1 - q_j^k) e^{-t_{ij}\lambda_j^k} \frac{(t_{ij}\lambda_j^k)^{y_{ij}}}{y_{ij}!} + q_j^k I(y_{ij} = 0) \quad (4) \\ &= \begin{cases} (1 - q_j^k) e^{-t_{ij}\lambda_j^k} \frac{(t_{ij}\lambda_j^k)^{y_{ij}}}{y_{ij}!} & \text{if } y_{ij} > 0 \\ (1 - q_j^k) e^{-t_{ij}\lambda_j^k} + q_j^k & \text{if } y_{ij} = 0 \end{cases} \end{aligned}$$

In particular, this equation provides the estimated *mean* or *expected value*,  $\mu_j^k$ , of the offense count  $Y_{ij}$  at age  $j$  of an individual  $i$  who is known to be in class  $k$ :

$$\mu_j^k = \mathbf{E}(Y_{ij} \mid \text{individual } i \text{ is in class } k) = (1 - q_j^k)(t_{ij}\lambda_j^k). \quad (5)$$

## 2.2 The Predictor Functions

It remains to model the unknown parameters  $\lambda_j^k$  and  $q_j^k$ . The  $\lambda_j^k$  are modeled by either the quadratic predictor functions:

$$\log(\lambda_j^k) = \beta_{0k} + \beta_{1k}j + \beta_{2k}j^2 \quad (6a)$$

or the cubic predictor functions:

$$\log(\lambda_j^k) = \beta_{0k} + \beta_{1k}j + \beta_{2k}j^2 + \beta_{3k}j^3, \quad (6b)$$

where for each latent class  $k$ , the unknown values  $\beta_{ik}$  need to be estimated.

The  $q_j^k$  model the amount of zero inflation, that is, the excess probability of individual

$j$  being in a “non-criminal” state. They are modeled by predictor functions given as either logit-linear:

$$\text{logit}(q_j^k) \equiv \log\left(\frac{q_j^k}{1 - q_j^k}\right) = \alpha_{0k} + \alpha_{1k}j \quad (7a)$$

or logit-quadratic:

$$\text{logit}(q_j^k) \equiv \log\left(\frac{q_j^k}{1 - q_j^k}\right) = \alpha_{0k} + \alpha_{1k}j + \alpha_{2k}j^2 \quad (7b)$$

or proportional to  $\log(\lambda_j^k)$  as in the ZIP( $\tau$ ) model of Lambert (2002):

$$\text{logit}(q_j^k) \equiv \log\left(\frac{q_j^k}{1 - q_j^k}\right) = -\tau_k \log(\lambda_j^k), \quad (7c)$$

where the  $\alpha_{ik}$  or  $\tau_k$  are again unknown values to be estimated.

Combining these equations, the overall likelihood function is given by

$$L_K(\theta) = \prod_{i=1}^N \sum_{k=1}^K p_k \prod_{j=L}^U \left[ (1 - q_j^k) e^{-t_{ij} \lambda_j^k} \frac{(t_{ij} \lambda_j^k)^{y_{ij}}}{y_{ij}!} + q_j^k I(y_{ij} = 0) \right], \quad (8)$$

where  $\theta$  is a vector consisting of all of the unknown parameters (i.e., of all of the  $\{p_k\}$ ,  $\{\beta_{ik}\}$ , and  $\{\alpha_{ik}\}$  or  $\{\tau_k\}$  as appropriate depending on which versions of the predictor functions (6a)–(6b) and (7a)–(7c) are used). Thus, the length  $f_K$  of the vector  $\theta$  is equal to  $7K - 1$  for the quadratic-quadratic pure-ZIP model (i.e., using equations (6a) and (7b)), or  $6K - 1$  for the cubic-ZIP( $\tau$ ) model (i.e., using equations (6b) and (7c)), etc. (The “ $-1$ ” comes because we omit  $p_K$  from the vector  $\theta$ , since we must have  $p_K = 1 - p_1 - p_2 - \dots - p_{K-1}$ .)

### 2.3 Fitting the Model

To fit the model, we require a maximum likelihood estimator (MLE), that is, a vector  $\hat{\theta}$  of estimated values of all the unknown parameters that maximizes the likelihood function  $L_K(\theta)$  in equation (8). Such  $\hat{\theta}$  consists of estimated values  $\{\hat{p}_k\}$ ,  $\{\hat{\beta}_{ik}\}$ , and  $\{\hat{\alpha}_{ik}\}$  or  $\{\hat{\tau}_k\}$

as appropriate, which in turn implies estimated values  $\{\hat{\lambda}_j^k\}$  and  $\{\hat{q}_k\}$ , and thus allows us to estimate all relevant quantities and probabilities associated with the model with  $K$  latent classes.

After obtaining the estimator  $\hat{\theta}$ , we can then use (8) to estimate the *a posteriori* probability  $\pi_i^k$  that individual  $i$  belongs to group  $k$ , by

$$\hat{\pi}_i^k = \frac{\hat{p}_k \prod_{j=L}^U \left[ (1 - \hat{q}_j^k) e^{-\hat{\lambda}_j^k} \frac{(\hat{\lambda}_j^k)^{y_{ij}}}{y_{ij}!} + \hat{q}_j^k I(y_{ij} = 0) \right]}{\sum_{m=1}^K \hat{p}_m \prod_{j=L}^U \left[ (1 - \hat{q}_j^m) e^{-\hat{\lambda}_j^m} \frac{(\hat{\lambda}_j^m)^{y_{ij}}}{y_{ij}!} + \hat{q}_j^m I(y_{ij} = 0) \right]}. \quad (9)$$

If we wish, we can also identify the most probable latent class of each individual  $i$ , as

$$\text{most probable latent class of individual } i = \arg \max_k \hat{\pi}_i^k. \quad (10)$$

We can then classify the individuals by most probable latent class and investigate the criminal characteristics of each such group.

Now, the length  $f_K$  of the vector  $\theta$  grows fairly quickly with  $K$ . So, for even moderately large  $K$ , computing the MLE is computationally challenging. Such issues are considered further below.

## 2.4 Prediction for New Subjects

This model also provides a method for *predicting* offense trajectories of new individuals. Indeed, if an  $(N+1)^{\text{st}}$  individual comes along, with offense data counts  $y_{N+1,L}, y_{N+1,L+1}, \dots, y_{N+1,U}$ , we could estimate the class membership probabilities  $\hat{\pi}_{N+1}^k$  from (9). Then, combining the values  $\hat{\pi}_{N+1}^k$  with the conditional means given by (5), we conclude that

$$\hat{y}_{N+1,j} \equiv \text{estimated mean of } Y_{N+1,j} = \sum_{k=1}^K \hat{\pi}_{N+1}^k (1 - q_j^k) (\lambda_j^k). \quad (11)$$

If  $\hat{y}_{N+1,j}$  is close to  $y_{N+1,j}$ , this indicates that the model does a good job of predicting future individuals' offense patterns. Such issues will be important in Section 3.2 below.

### 3 Number of Latent Classes

The above discussion provides a complete and unambiguous recipe for estimating the entire multidimensional vector of parameters  $\theta$ , as well as the *a posteriori* group membership probabilities  $\pi_i^k$  in (9), at least once we have specified the number  $K$  of latent classes. However, the choice of number of classes  $K$  must first be made.

#### 3.1 BIC and AIC

A conventional method for selecting the number of classes, or more generally the number of parameters to use in a model, is to minimize the Bayesian Information Criterion (BIC), given by

$$\text{BIC} = -2 \log(\text{likelihood}) + (\text{number of free parameters}) \log(\text{number of observations}). \quad (12)$$

For the model described in Section 2.1, the likelihood function is  $L_K(\theta)$ , and the number of free parameters is equal to the length  $f_K$  of the vector  $\theta$  and hence is equal to  $7K - 1$  for the quadratic-quadratic model, or  $5K - 1$  for the cubic-tau model, and so on. Also, the number of observations is taken to be  $N(U - L + 1)$ , the total number of  $y_{ij}$  data values. (Note that some implementations by default use  $N$  instead of  $N(U - L + 1)$ . For consistency we stick with  $N(U - L + 1)$  here, but in any case such difference will not greatly affect the results.) Hence, the formula becomes

$$\text{BIC} = -2 \log(L_K(\hat{\theta})) + f_K \log(N(U - L + 1)). \quad (13)$$

The BIC criterion would then have us choose the value of  $K$  that minimizes (12).

Closely related to BIC is the Akaike information criterion (AIC), proposed by Akaike (1974), defined by

$$\text{AIC} = -2 \log(\text{likelihood}) + 2(\text{number of free parameters}). \quad (14)$$

which in our case becomes

$$\text{AIC} = -2 \log(L_K(\hat{\theta})) + 2f_K. \quad (15)$$

The AIC criterion would then have us choose the value of  $K$  that minimizes (15). A comparison of (14) and (12) (or of (15) and (13)) shows that AIC is similar in spirit to BIC, and indeed we shall see below that it appears to have similar difficulties and limitations to BIC.

We note that BIC is sometimes defined as  $-1/2$  times the value described in (12) and (13) (and similarly for AIC). In this case, the criterion would instead have us choose the value of  $K$  that *maximizes* the BIC. So, to compare our results with other results using the alternative definition, simply multiply all of our BIC (and AIC) values by  $-1/2$ .

Below, we shall consider both BIC and AIC, and will find (as previously observed) that neither provides a satisfactory resolution for determining the number of latent classes  $K$ . Because of these difficulties, we will consider an alternative method, cross-validation error (CVE), for determining the number of latent classes  $K$ , and will demonstrate below that CVE is superior to BIC or AIC for this purpose.

### 3.2 The Cross-Validation Approach

The previous section discussed the challenge of accurately computing the BIC. However, we shall argue below that even when computed correctly (e.g. with *crimCV*), BIC still does not provide a very useful criterion for determining the number  $K$  of latent classes. Thus, we

propose an alternative criterion for this purpose. Specifically, we propose to make use of *cross-validation* (Stone, 1974). This is a method of using data to test the predictive power of a model. If the model predicts well, then this suggests that it is a good and appropriate model which provides a good “fit” of the data and should be used.

In the present context, we proceed as follows. For each possible choice of number of latent classes  $K$ , we compute a cross-validation error (CVE) indicating the extent to which the model fails to perfectly model the data. The final choice of  $K$  is then the one that minimizes this CVE value.

More specifically, we use *leave-one-out* cross-validation. This method measures the accuracy of the fit for individual  $i$  by using estimates of the model parameters  $\theta$  based on data for all the *other* individuals but *not* individual  $i$ . Specifically, for a given choice of the number  $K$  of latent classes, we proceed as follows. For each individual  $i$ , we *omit* the data for individual  $i$ , and then determine an MLE  $\hat{\theta}^{[-i]}$  for  $\theta$  based on data for the *other*  $N - 1$  individuals only. Then, using the parameter values  $\hat{\theta}^{[-i]}$ , we then use (9) and (11) to obtain estimates  $\hat{y}_{ij}^{[-i]}$  for the various  $y_{ij}$  values ( $L \leq j \leq U$ ). That is, we obtain the best possible estimates for the offense counts for individual  $i$ , using a model whose parameters have been fit *without* using the offense data of individual  $i$ . This provides a fair measure of the accuracy of the model, without encouraging over-fitting (see e.g. Day et al., 2007).

Once we have estimates  $\{\hat{y}_{ij}^{[-i]}\}_{j=L}^U$ , then the cross-validation error for individual  $i$ ,  $\text{CVE}(i)$ , can be measured in terms of the average absolute difference between the true values  $(y_{i,L}, y_{i,L+1}, \dots, y_{i,U})$  and the predicted values  $(\hat{y}_{i,L}^{[-i]}, \hat{y}_{i,L+1}^{[-i]}, \dots, \hat{y}_{i,U}^{[-i]})$ , i.e.

$$\text{CVE}(i) = \frac{1}{U - L + 1} \sum_{j=L}^U |y_{ij} - \hat{y}_{ij}^{[-i]}|.$$

By repeating this process for each of the  $N$  individuals  $i$ , and then averaging the cross-

validation errors  $\text{CVE}(i)$  over all  $N$  individuals, we get our final cross-validation error (CVE):

$$\text{CVE} = \frac{1}{N} \sum_{i=1}^N \text{CVE}(i).$$

The value CVE thus provides a fair measure of how appropriate the chosen group number  $K$  is for the given data in terms of how accurately a model with that number of groups is able to predict the offender data of new individuals. If CVE is large, this indicates that the model with  $K$  groups is not a good statistical model for this data. By contrast, if CVE is small, then the model with  $K$  groups is doing a good job of predicting offender data of new individuals.

The cross-validation criterion for number of groups then involves simply choosing the value of  $K$  that minimizes CVE. The free software package *crimCV* computes CVE for any criminal offense data and any number  $K$  of latent groups, and is thus appropriate for applying this cross-validation criterion. (Another possible method is the bootstrap likelihood ratio test (BLRT), not considered here; see Nylund, Asparouhov & Muthén, 2007; Kreuter & Muthén, 2008.) We shall see in Section 6 that the cross-validation criterion usually provides a sensible, stable recommendation for the number of latent classes  $K$ , and thus largely avoids the problems and limitations of BIC and AIC.

## 4 Toronto Juvenile Offender Samples (TO1 and TO2)

To study this model, and the *Proc Traj* and *crimCV* software, and the BIC and CVE criteria, we consider two Toronto juvenile offender data sets, referred to as TO1 ( $N = 378$ ) and TO2 ( $N = 386$ ), previously studied by Day, Bevc, Duchesne, Rosenthal, Rossman, and Theodor (2007) (see also Day, Bevc, Rosenthal, Duchesne, Rossman, & Theodor, 2003; Day, Bevc, Duchesne, Rosenthal, Sun, & Theodor, 2008; Day, Nielsen, Ward, Sun, Rosenthal, Duchesne, Bevc, & Rossman, 2011; Ward et al., 2010). This data is publicly available for inspection

in completely de-identified form, by contacting I. Bevc or by installing the *crimCV* software package discussed below.

All the youths in this study had served a sentence between January 1, 1986 and December 30, 1997 at one of two open custody facilities (i.e., group homes) for juvenile offenders in Toronto, Canada, operated by a children's mental health centre. During this period, a total of 764 male offenders served a sentence at one of the two sites; therefore, our research involves the entire population of youth from these facilities during this period. Information about the two samples is presented in Table 1.

**[INSERT TABLE 1 ABOUT HERE.]**

The first sample, "TO1", comprised a randomly selected sample of 378 youth, from the population of youth at these facilities during this period. The sample was, on average, 17.6 years at the time of admission into the facility and the average sentence length was 124.6 days (Median = 92 days). Their criminal activity was tracked for an average of 18.7 years (range = 12.3 – 29.3 years), from their first recorded involvement with the justice system to September 26, 2007. The average age at the end of the follow-up was 34.1 years (range = 28.7 – 40.5 years).

The remaining 386 offenders from this population constituted the second sample, "TO2". This group was, on average, 17.7 years at the time of admission into the facility and the average sentence length was 122.6 days (Median = 93 days). Their criminal activity was tracked for an average of 16.4 years (range = 9.8 – 28.7 years), from their first recorded involvement with the justice system to September 26, 2007. Their average age at first court contact was 15.6 years (range = 9.6 – 19.4 years) and the average age at the end of the follow-up was 32.0 years (range = 26.3 – 40.2 years). A strength of these samples is that the follow-up periods extended from late childhood (for offenses committed under the Juvenile Delinquents Act [JDA] in Canada, for which the minimum age of criminal liability was 7

years, unlike the Young Offenders Act [YOA], to which most of the criminal data apply, and the current Youth Criminal Justice Act [YCJA], for which the minimum age is 12 years) and early adolescence into adulthood, well beyond the challenging period of emerging adulthood of the early 20's (Arnett, 2000, 2007).

## 4.1 Criminal Data

The criminal data for TO1 were received, on March 17, 2001 and then on September 26, 2007, from four sources: (1) the (Ontario) Ministry of Community and Social Services (MCSS); (2) the (Ontario) Ministry of Community Safety and Correctional Services (MCSCS); (3) the Canadian Police Information Centre (CPIC); and (4) the predisposition reports (PDRs) maintained by the children's mental health centre. The criminal data for TO2 were received from three sources: (1) the (Ontario) Ministry of Community Safety and Correctional Services (MCSCS); (2) the Canadian Police Information Centre (CPIC); and (3) the predisposition reports (PDRs) maintained by the children's mental health centre. These data sources were used to ensure a high degree of completeness and accuracy of the information, which is essential for research that requires an accurate temporal sequencing of criminal activity (Smith, Smith, & Norma, 1984).

The reason only three data sources were used for TO2 (as well as for the second follow-up for TO1 as noted below) was that, at some point between March 17, 2001 and September 26, 2007, the MCSS system was integrated into the MCSCS and thus two sources became one main source. Due to the nature of the data integration process and the importance of continued monitoring of active offenders, historical data for juvenile offenders in Phase 1 (i.e., ages 12- 15 years) were available only for those offenders who were still actively serving youth sentences at the time of our inquiry. Thus, due to the limitations of accessible data at the time, the majority of Phase 1 data for TO2 were retrieved through PDRs.

As well, to aid the accuracy of our time-at-risk adjustments (see below), supplemental movement data containing (institutional) location start and exit dates were provided by the

MCSCS for our data requested on September 26, 2007. Due to the implementation of a new criminal record monitoring system between the first and second follow-up periods, TO2 data were received in electronic format from the Ministry and contained more information than that received on the paper profiles (i.e. "rap sheets") for the initial follow-up of TO1. Not only could the charges and dates of convictions be discerned as before, but now the movement data associated with each charge was provided and allowed for a more accurate account of time-served per date of conviction. As a result of these differences in data received between the two follow-up periods, we decided not to combine the data for TO1 and TO2 into one large data set for analyses. The criminal data for these samples gave us year-by-year offense counts  $\{y_{ij}\}$ ; for TO1 we have  $1 \leq i \leq N = 378$  and  $L = 8 \leq j \leq U = 38$ , while for TO2 we have  $1 \leq i \leq N = 386$  and  $L = 9 \leq j \leq U = 38$ .

## 4.2 Time-at-Risk Adjustments

As described in Day et al. (2007), the time-at-risk adjustment contained additional challenges since the data included dates of court contact but not dates of offense. This necessitated computing *estimates*  $t_{ij}$  of the fraction of the year at age  $j$  when subject  $i$  was at large. These  $t_{ij}$  values can then be used as offsets as in the previous section (as in Piquero et al., 2001, p 59). Unfortunately, *Proc Traj* cannot easily handle time offsets. Alternatively, we can use the "divide and round" (DAR) approximation of Ward et al. (2010) (see also Eggleston et al., 2004, p. 6 bottom), consisting of adjusting each count  $y_{ij}$  by dividing it by the corresponding exposure time  $t_{ij}$  and rounding the result to the nearest integer (truncated at a maximum of 25). We feel that time offsets are a far preferable method, since DAR does not distinguish between, for example, "5 offenses in 3 months at large" and "20 offenses this year" even though the effect of these two different realities on our conclusions should be far from identical. However, DAR does provide a clear and unbiased data set with which to compare *Proc Traj* to other software, as we do in the next section.

## 5 Software Considerations

As noted above, the computation of the MLE for these models is very challenging, especially for larger numbers of latent classes. To compute the MLE, we originally used the SAS procedure *Proc Traj* provided by Jones (2001; see also Jones et al., 2001; Jones, & Nagin, 2007). However, we found that *Proc Traj* sometimes failed to converge consistently, and sometimes gave different values when used with different versions of the program or different initialisations, and sometimes failed to find the true MLE (see Section 5.2).

Another commonly-used software package is *Mplus* ([www.statmodel.com](http://www.statmodel.com); Muthén and Muthén, 2001), which provides some improvements over *Proc Traj*, but still sometimes fails to find the true MLE (see Section 5.2). In addition, both *Proc Traj* and *Mplus* are proprietary packages which cost money and unfortunately do not allow for inspection and modification of the source code. For all these reasons, we eventually decided to write our own software package *crimCV*, described next.

### 5.1 A New Software Package: *crimCV*

Our new software package *crimCV* incorporates the above ZIP models for criminal offender data, computes the MLE (and BIC and AIC) more reliably than *Proc Traj*, and also computes the quantity CVE. Furthermore, it is available as a free, publicly available package to augment the free statistical software R, at no cost and with full access to the source code.

To run *crimCV*, one should proceed as follows:

- 1) Obtain the R programming environment by following the instructions at:

<http://www.r-project.org/>

- 2) Run R, by double-clicking the R icon or typing 'R' at the terminal.
- 3) At the R prompt '>', type 'chooseCRANmirror()' and select the closest repository.
- 4) At the R prompt '>', type 'install.packages("crimCV")', which will install the *crimCV* package. (Note: you need to have administrative privileges on your computer to

install packages.)

5) At the R prompt '>', type `'help("crimCV")'` for a description of the software, options and an example of usage.

After following the above steps, *crimCV* can be freely run, both on the TO1 and TO2 data to check our results herein, and on any other criminal offense data as desired.

## 5.2 Comparison of the Different Software Packages

To compare these various software packages, we had each of *Proc Traj* and *Mplus* and *crimCV* maximize the log-likelihood and compute the corresponding BIC and AIC values for the TO1 and TO2 data sets. Since *Proc Traj* cannot easily handle time offsets, nor was it designed for the ZIP( $\tau$ ) model, we therefore focused on the quadratic-quadratic pure-ZIP model together with the DAR method of correcting for time-at-risk.

The computed results are presented in Table 2.

[INSERT TABLE 2 ABOUT HERE.]

These results show that for small numbers of latent groups, each of *Proc Traj* and *Mplus* and *crimCV* obtain the same values for the maximum log-likelihood (and hence for BIC and AIC, as well), as we would hope. However, for larger numbers of latent groups, discrepancies start to arise. In such cases, the *crimCV* values for the log-likelihood are consistently *larger* (i.e., less negative) than those computed by the other software packages. This shows that it is *Proc Traj* and *Mplus* that sometimes fail to *maximize* the log-likelihood, and thus obtain too small an estimate of the log-likelihood and thus a correspondingly too small estimate of the values of BIC and AIC.

We note that this run used the *Proc Traj* default starting values. If *Proc Traj* is instead initialised with excellent starting values as obtained from *crimCV* then it will have a better

chance of computing the correct answers; however such excellent starting values would not normally be available to the user.

Similarly, *Mplus* was run with the command “start = 100 10;”. This means that in the initial stage, 100 random sets of starting values were generated. An optimization is then carried out for ten iterations using each of the random sets of starting values. The ending values from the optimizations with the 10 highest loglikelihoods are then used as the starting values in the final stage optimizations (EM algorithm in our case). Note that the default starting values setting in *Mplus* for mixture models is 10 2 (10 random sets in the first stage and 2 in the final stage), but we increase it to 100 10 (which is suggested by the program for  $K > 2$ ) to improve the *Mplus* results. It is possible that increasing these values still larger would improve *Mplus*’s performance somewhat, but at the expense of even greater computation time.

We conclude that *Proc Traj* and *Mplus* do not always accurately maximize the likelihood nor compute correct values for BIC and AIC. They are thus sub-optimal as software for optimizing the number of latent classes. (This is not intended as a criticism of *Proc Traj* and *Mplus*, since such high-dimensional maximisation is notoriously difficult, but it does illustrate the limitations of the existing software.)

By contrast, *crimCV* more accurately maximizes the likelihood and computes the BIC (and AIC) values. In addition, *crimCV* can directly handle time-at-risk offset values  $t_{ij}$ , as well as both pure-ZIP and ZIP( $\tau$ ) models. Furthermore, it can also compute the CVE, as discussed further below. In addition, it is freely available including its source code which can be inspected and modified as desired.

## 6 Comparison of AIC, BIC, and CVE

We have already noted in the introduction that BIC has been found to be problematic in many applications. Of course, some of these difficulties may have been due to the compu-

tational limitations of *Proc Traj* as noted above. However, we now argue that even when computed correctly (e.g. with *crimCV*), BIC still does not provide a very useful criterion for determining the number  $K$  of latent classes, and CVE is, in fact, a superior method. We illustrate this by applying BIC and CVE (and AIC) to our two offender data sets, and find that CVE provides results which suggest more reasonable numbers of latent classes.

## 6.1 Optimal Number of Groups for the Toronto Data

We apply our *crimCV* software to the two Toronto data sets, TO1 and TO2, with what we consider to be the most appropriate model, namely the cubic-ZIP( $\tau$ ) model (i.e., using equations (6b) and (7c)), treating the time-at-risk exposure times  $t_{ij}$  as offsets in our model rather than incorporating them using the questionable DAR method. Our results are presented in Table 3.

[INSERT TABLE 3 ABOUT HERE.]

Table 3 shows that BIC and AIC decrease seemingly without limit, thus always advocating more and more latent groups. This mirrors the problems reported previously in the literature wherein BIC advocates more groups until such time as the computational software (e.g. *Proc Traj*) crashes and cannot compute larger numbers of groups.

By contrast, CVE advocates a more reasonable numbers of groups since it achieves its minimum value at a plausible number of groups, namely 8 for TO1, and 7 for TO2. Thus, this provides some evidence that CVE is a useful criterion for determining the number of latent classes  $K$ , more so than is BIC or AIC.

## 6.2 Latent Group Analysis

We have thus used *crimCV* to determine the optimal number of latent groups according to the CVE criterion, namely 8 for TO1, and 7 for TO2.

As described in Day et al. (2011; see also Day, Nielsen, Ward, Rosenthal, Sun, Bevc, Duchesne, Rossman, & Samuels, 2010), these numbers of groups then lead to appropriate interpretations of the criminal behavior in the resulting latent classes. For the TO1 data, the groups were heuristically labelled Low Rate Desister (LRD), comprising 28.0% of the sample, Low Rate Chronic (LRC), comprising 26.2% of the sample, Low Rate Adolescent Peaked (LRADOLP), comprising 16.4% of the sample, Moderate Rate Chronic II (MRC-II), comprising 11.9% of the sample, Moderate Rate Chronic I (MRC-I), comprising 5.3% of the sample, High Rate Adolescent Peaked (HRADOLP), comprising 4.8% of the sample, Moderate Rate Adult Peaked (MRADLP), comprising 4.5% of the sample, and Moderate Rate Escalator (MRE), comprising 2.9% of the sample. None of the groups had less than 10 individuals in them and the mean posterior probability coefficients were quite high across all four groups, exceeding .88. The trajectory groups are plotted in Figure 1.

**[INSERT FIGURE 1 ABOUT HERE.]**

Comparisons on offending-related variables across the seven groups indicated that the LRD group had the latest age of first court contact ( $M = 15.8$  years), the earliest age of last court contact ( $M = 20.5$  years), and the shortest criminal career, spanning, on average, 4.2 years. The HRADOLP group had the earliest age of first court contact ( $M = 13.7$  years). Not surprisingly, the MRE had the latest age of last court contact ( $M = 34.0$  years) and the longest criminal career at 19.3 years, on average. The MRC I group had the second longest criminal career at 17.8 years, on average, followed by MRC II at 15.9 years, on average, and the LRC group at 13.5 years, on average. Over the duration of their criminal trajectories, the MRE group amassed the largest number of court contacts (adjusted by time-at-risk), with 110.1, on average, followed by the HRADOLP group with 89.5, on average, and the MRADOLP group, 56.4, on average. By contrast, the LRD group had the fewest court contacts, on average, with 4.6.

Similar to the TO1 data, analysis of the TO2 data yielded seven trajectory groups. These groups were heuristically labeled Moderate Rate Chronic I (MRC I), comprising 3.6% of the sample; High Rate Adult Peaked (HRADLP), comprising 3.9% of the sample; High Rate Adolescence Peaked (HRADOLP), comprising 4.4% of the sample; Moderate Rate Adolescence Peaked (MRADOLP), comprising 11.7% of the sample; Moderate Rate Chronic II (MRC II), comprising 14.2% of the sample; Low Rate Desister (LRD), comprising 29.8% of the sample; and Low Rate Chronic (LRC), comprising 32.4% of the sample. None of the groups had less than 10 individuals in them and the mean posterior probability coefficients were quite high across all four groups, exceeding .89. These trajectories are plotted in Figure 2.

**[INSERT FIGURE 2 ABOUT HERE.]**

Comparisons on offending-related variables across the seven groups revealed some interesting similarities with the TO1 data. The LRD group had the latest age of first court contact ( $M = 16.4$  years), the earliest age of last court contact ( $M = 19.5$  years), and the shortest criminal career, spanning, on average, 3.1 years (see Figure 1). Once again, the HRADOLP group had the earliest age of first court contact ( $M = 14.3$  years). The MRC I group had the latest age of last court contact ( $M = 31.9$  years) and the longest criminal career at 16.6 years, on average. The MRC II group had the second longest criminal career at 14.8 years, on average. Over the duration of their criminal trajectories, the HRADLP group amassed the largest number of court contacts (adjusted by time-at-risk), with 78.1, followed by the HRADOLP group with 62.7, on average, and the HRC group with 52.1, on average. By contrast, the LRD group had the fewest court contacts, on average, with 4.8.

## 7 Discussion

This paper has argued that the traditional method of computing BIC using *Proc Traj* or *Mplus*, does not maximize the likelihood in a consistently accurate manner and thus cannot be completely trusted to compute BIC. Inspired by this, we have presented the free software package *crimCV* for accurately maximizing likelihood and computing BIC (and AIC). *crimCV* has the additional advantages that it can handle ZIP( $\tau$ ) as well as pure-ZIP models and can incorporate time-at-risk  $t_{ij}$  as a true offset variable (rather than only with the more questionable method of dividing-and-rounding (DAR) the offense count data). It can also compute the cross-validation error (CVE) in addition to BIC and AIC.

This paper has further argued that BIC, the traditional method of estimating number of latent classes (groups) when analyzing criminal offense trajectories, is fundamentally flawed since it often increases monotonically with group number – thus arguing for larger and larger numbers of groups, which eventually have to be cut off due to computational or other *ad hoc* reasons. By contrast, we have argued that the cross-validation error (CVE) is a theoretically sound method of evaluating model appropriateness by measuring predictive ability, which makes sense logically and which also provides appropriate group number recommendations in practice.

We applied our *crimCV* software to two Toronto data sets of juvenile offenders. In both cases, CVE has provided superior performance to that of BIC (or AIC), by achieving its minimum at a plausible number of latent groups (8 and 7, respectively), in contrast to BIC and AIC which continue to decrease without apparent limit. We view this as convincing evidence that cross-validation is a clear, fair, objective, and effective method of determining the number of latent classes, and should be preferred over conventional methods such as BIC or AIC.

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**Table 1.** Mean (SD) characteristics of the TO1 and TO2 juvenile offender datasets.

| Variable                         | TO1 ( $N = 378$ )  | TO2 ( $N = 386$ ) |
|----------------------------------|--------------------|-------------------|
| End of Follow-up Period:         | Sept. 26, 2007     | Sept. 26, 2007    |
| Age at admission into youth home | 17.6 years (.85)   | 17.7 years (1.0)  |
| Sentence length at youth home    | 124.6 days (109.8) | 122.6 days (95.6) |
| Length of follow-up              | 18.7 years (3.0)   | 16.4 years (4.1)  |
| Age at first court contact       | 15.5 years (1.8)   | 15.6 years (1.6)  |
| Age at last court contact        | 26.1 years (5.5)   | 24.6 years (5.2)  |
| Age at end of follow-up          | 34.1 years (2.6)   | 32.0 years (4.0)  |
| Trajectory length                | 10.7 years (5.6)   | 9.5 years (5.6)   |

**Table 2.** Computations of the maximum log-likelihood, AIC, and BIC, using each of the software packages *Proc Traj*, *Mplus*, and *crimCV*, on the TO1 and TO2 juvenile offender datasets, for various numbers of latent groups (*ngr*). For small *ngr*, the three software packages tend to agree. However, for larger numbers of groups, *crimCV* consistently finds larger (less negative) maximum log-likelihood values, and correspondingly smaller AIC and BIC values, than do *Proc Traj* and *Mplus* (and in some cases, *Proc Traj* fails to converge entirely). This illustrates that *crimCV* does a superior job of maximising the log-likelihood function.

**TO1:**

| ngr | llike            |              |               | AIC              |              |               | BIC              |              |               |
|-----|------------------|--------------|---------------|------------------|--------------|---------------|------------------|--------------|---------------|
|     | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> |
| 1   | -13756           | -13756       | -13756        | 27524            | 27524        | 27524         | 27568            | 27568        | 27568         |
| 2   | -13756           | -12133       | -12133        | 27524            | 24293        | 24293         | 27568            | 24389        | 24389         |
| 3   | -11604           | -11604       | -11555        | 23249            | 23249        | 23150         | 23396            | 23396        | 23298         |
| 4   | -11254           | -11295       | -11254        | 22561            | 22644        | 22561         | 22760            | 22843        | 22760         |
| 5   | -11106           | -11123       | -11062        | 22281            | 22314        | 22192         | 22531            | 22565        | 22443         |
| 6   | -11051           | -10950       | -10934        | 22184            | 21981        | 21949         | 22486            | 22347        | 22251         |
| 7   | -10880           | -10904       | -10846        | 21857            | 21904        | 21788         | 22210            | 22126        | 22141         |
| 8   | -11229           | -10811       | -10757        | 22568            | 21731        | 21624         | 22973            | 22137        | 22029         |
| 9   | failed           | -10770       | -10711        | failed           | 21663        | 21549         | failed           | 22120        | 22003         |

**TO2:**

| ngr | llike            |              |               | AIC              |              |               | BIC              |              |               |
|-----|------------------|--------------|---------------|------------------|--------------|---------------|------------------|--------------|---------------|
|     | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> | <i>Proc Traj</i> | <i>Mplus</i> | <i>crimCV</i> |
| 1   | -11239           | -11239       | -11239        | 22490            | 22490        | 22490         | 22535            | 22535        | 22535         |
| 2   | -10119           | -10119       | -10108        | 20264            | 20264        | 20241         | 20359            | 20359        | 20337         |
| 3   | -9613            | -9572        | -9572         | 19266            | 19183        | 19183         | 19413            | 19331        | 19331         |
| 4   | -9619            | -9387        | -9387         | 19292            | 18829        | 18829         | 19491            | 19028        | 19028         |
| 5   | -9341            | -9252        | -9236         | 18749            | 18571        | 18540         | 18999            | 18821        | 18790         |
| 6   | -9183            | -9184        | -9149         | 18448            | 18451        | 18381         | 18750            | 18752        | 18682         |
| 7   | failed           | -9068        | -9064         | failed           | 18232        | 18224         | failed           | 18585        | 18577         |
| 8   | failed           | -9038        | -8980         | failed           | 18185        | 18070         | failed           | 18590        | 18475         |
| 9   | failed           | -8951        | -8928         | failed           | 18026        | 17980         | failed           | 18482        | 18437         |
| 10  | failed           | -8937        | -8901         | failed           | 18011        | 17940         | failed           | 18519        | 18448         |

**Table 3.** Maximum log-likelihood, and the three optimality criteria (AIC, BIC, and CVE), for the datasets TO1 and TO2, for various numbers of latent groups. The smallest value for each of the three criteria is written in **boldface**. In particular, note that AIC and BIC each continue to shrink and thus advocate more and more latent groups without limit, while CVE reaches a minimum value and then increases again. (We end the runs after 9 and 8 groups, respectively, since after that the models *saturate* with one of the group probabilities becoming zero, so there are no further meaningful results to be obtained.)

**TO1:**

| ngr | llike     | AIC             | BIC             | CVE              |
|-----|-----------|-----------------|-----------------|------------------|
| 1   | -13967.63 | 27945.26        | 27982.26        | 1.0902792        |
| 2   | -11929.40 | 23880.81        | 23962.22        | 0.9128347        |
| 3   | -11424.68 | 22883.37        | 23009.18        | 0.9592355        |
| 4   | -11191.28 | 22428.55        | 22598.77        | 0.9052791        |
| 5   | -11016.19 | 22090.37        | 22304.99        | 0.8535441        |
| 6   | -10886.30 | 21842.61        | 22101.63        | 0.8334242        |
| 7   | -10805.59 | 21693.18        | 21996.60        | 0.8261734        |
| 8   | -10732.58 | 21559.16        | 21906.99        | <b>0.8123785</b> |
| 9   | -10684.54 | <b>21475.08</b> | <b>21867.31</b> | 0.8240060        |

**TO2:**

| ngr | llike      | AIC             | BIC             | CVE              |
|-----|------------|-----------------|-----------------|------------------|
| 1   | -11700.095 | 23410.19        | 23447.45        | 0.7731111        |
| 2   | -10183.745 | 20389.49        | 20471.47        | 0.6698342        |
| 3   | -9717.829  | 19469.66        | 19596.35        | 0.6529144        |
| 4   | -9541.180  | 19128.36        | 19299.76        | 0.6364674        |
| 5   | -9377.273  | 18812.55        | 19028.66        | 0.6171476        |
| 6   | -9302.665  | 18675.33        | 18936.16        | 0.6122683        |
| 7   | -9223.541  | 18529.08        | 18834.63        | <b>0.6108820</b> |
| 8   | -9165.115  | <b>18424.23</b> | <b>18774.49</b> | 0.6271084        |

**Figure 1.** Estimated Criminal Trajectories for the Eight-Group Model for TO1.

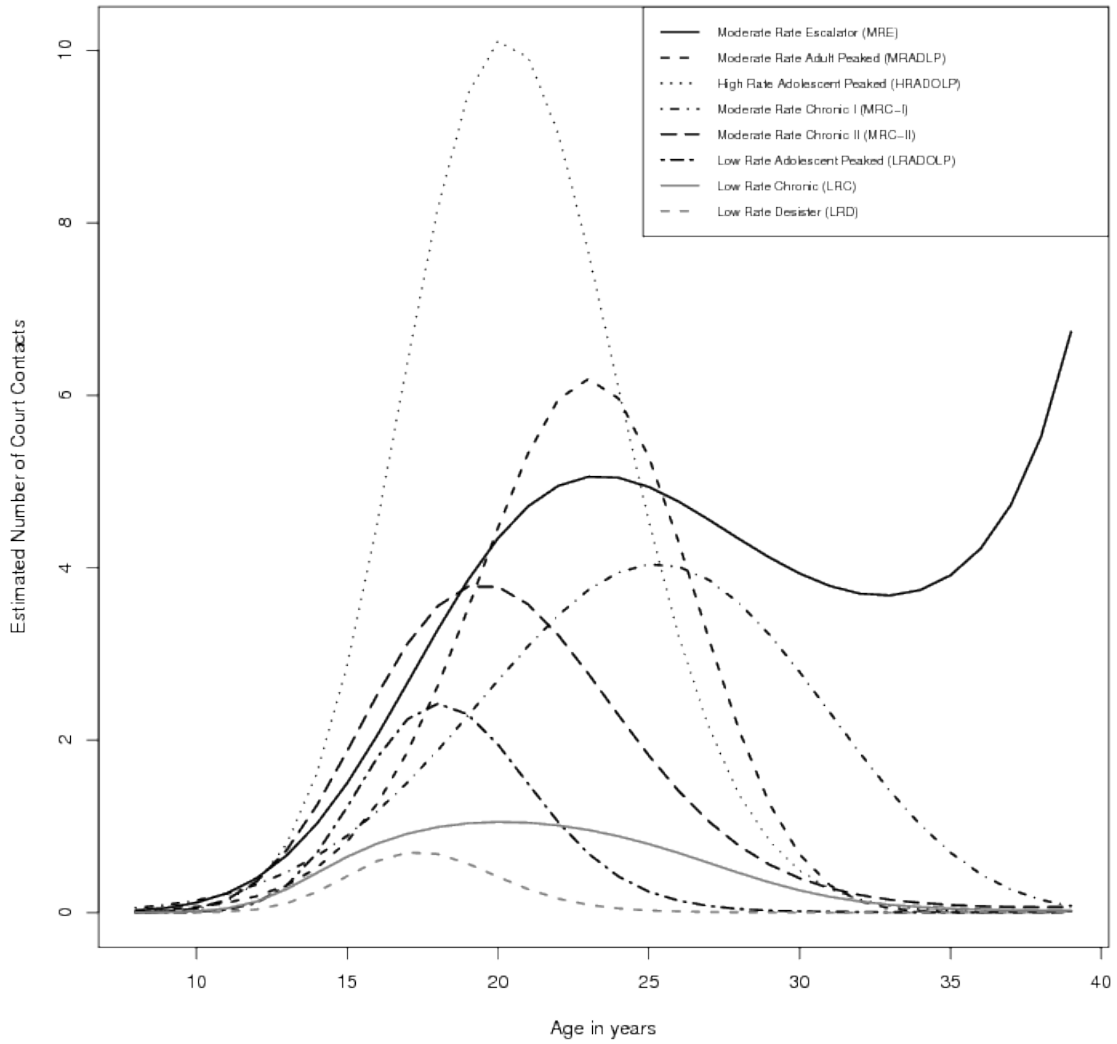


Figure 2. Estimated Criminal Trajectories for the Seven-Group Model for TO2.

