Errata for the SECOND EDITION of "A First Look at Rigorous Probability", by Jeffrey S. Rosenthal, World Scientific Publishing Co., 2006.

Errata to Fifth Printing, 2011:

[With thanks to Daniel Firka and Julian Ziegler Hunts.]

- Page 25, line 1: "setset" should be "subset".
- Page 35, line 4: "considering" should be "consider".
- Page 36, line 9: missing "}".
- Page 36, several places: omit extra $\{\ldots\}$, e.g. "$\{H_n \cap H_{n+1}\}$" should be simply "$H_n \cap H_{n+1}$", etc.
- Page 39, Exercise 3.6.5: for clarity, assume $\mathcal{F} = 2^\Omega$.
- Page 39, line 4: "P" should be "$\mathbf{P}$".
- Page 40, Exercise 3.6.11: the notation "$\sim$" has not yet been introduced, so "$X_n \sim \text{Uniform}(\{1, 2, \ldots, n\})$" could be replaced by "$\mathbf{P}(X_n = i) = 1/n$ for $i = 1, 2, \ldots, n$".
- Page 40, Exercise 3.6.13: should be moved to LATER (since it uses expectation), e.g. as Exercise 4.5.16. (And "E" should be "$\mathbf{E}$", twice.) It could be replaced by e.g.

  Let $X_1, X_2, \ldots$ be defined jointly on some probability space $(\Omega, \mathcal{F}, P)$, with $\sum_{i=1}^{\infty} i^2\, \mathbf{P}(i \leq X < i+1) \leq C < \infty$ for all i. Prove that $\mathbf{P}[X_n \geq n \ i.o.] = 0$.

- Page 46, statement of Theorem 4.2.2: assume the X_n are non-negative, and then omit "$\mathbf{E}(X_1) > -\infty$" (since we haven't yet defined expected values of general random variables).
- Page 46, lines -2 and -1: "E" should be boldface (twice).
- Page 49, exercise 4.3.3(a): change "Z^+ and Z^-" to "$Z^+ - Z^-$".
- Page 54, Exercise 4.5.13(d): replace "$\mathbf{E}(X) < \infty$" by "$0 < \mathbf{E}(X) < \infty$".
- Page 55, Exercise 5.5.9 Hint: specify that $y > 0$ for the first part, too.
- Page 58, Lemma 5.2.1: state it as "if and only if".
- Page 66, Exercise 5.5.13, Hint: "r different sums" should be "$r+1$ different sums".
- Page 71, Exercise 6.3.1 is a repeat of Exercise 4.5.1 (page 52), and should be replaced by e.g.:

Let μ have density $x^3 \mathbf{1}_{0<x<1}$, and let ν have density $x \mathbf{1}_{0<x<2}$.
(a) Compute $\mathbf{E}(X)$ where $Ł(X) = \frac{1}{3}\mu + \frac{2}{3}\nu$.
(b) Compute $\mathbf{E}(Y^2)$ where $Ł(Y) = \frac{1}{6}\mu + \frac{1}{3}\delta_2 + \frac{1}{2}\delta_5$.
(c) Compute $\mathbf{E}(Z^3)$ where $Ł(Z) = \frac{1}{8}\mu + \frac{1}{8}\nu + \frac{1}{4}\delta_3 + \frac{1}{2}\delta_4$.

- Page 76, Exercise 7.2.5(d): for clarity, prepend "Use the fact that $s(c) = 1$ to".
- Page 85, statement of Theorem 8.1.1, displayed eqn: "nu_0" should be "ν_0".
- Page 114, bottom line: "β^α" should be "$\beta^{-\alpha}$".
- Page 115, top line: "t^{x-1}" should be "$t^{\alpha-1}$".
- Page 125, middle of page, displayed equations: final inequality ($\leq$) is actually an inequality ($=$).
- Page 206 middle, "it suffice to assume" should be "it suffices to assume".

Errata to Fourth Printing, 2010 (to be corrected in Fifth Printing, 2011):

[With thanks to David Alexander, Martin Hazelton, Andrea Lecchini-Visintini, Gareth Roberts, Igal Sason, Mohsen Soltanifar, and Albert Zevelev.]

- p. 13, line 3 of proof of Lemma 2.3.11: "$\mathcal{B}_m^C$" should be "B_m^C".
- p. 23, Exercise 2.7.3, part (a): "an semialgebra" should be "a semialgebra".
- p. 31, line 5: "$f^{-1}((\infty, x])$" should be "$f^{-1}((-\infty, x])$".
- p. 44, line 1, and again on line 19: "$\sum_j y_j \mathbf{1}_{B_i}$" should be "$\sum_j y_j \mathbf{1}_{B_j}$".
- p. 48, line -5: "non non-negative" should be "not non-negative".
- p. 66, line -8: "Holder" should be "Jensen".
- p. 69, first line of Proposition 6.2.1: "μ_I" should be "μ_i".
- p. 74, line 7: "$F(z)$ for each $z \in \mathbf{R}$" should be "$F(x)$ for each $x \in \mathbf{R}$".
- p. 85, displayed equation in Theorem 8.1.1: "$X_= i_1$" should be "$X_1 = i_1$".
- p. 106, line 15: "integral" should be "integrable".
- p. 106, line 17: "$\mathbf{E}(X)$" should be "$\mathbf{E}(X_0)$".
- p. 106, line -1: "$E(Y)$" should be "$\mathbf{E}(Y)$".
- p. 136, Exercise 11.3.2: "Y_n" should be "Y_k" (twice).
- p. 137, conclusion of Theorem 11.4.1: "$\mu \Rightarrow \mu$" should be "$\mu_n \Rightarrow \mu$".

- p. 146, line −6: "mu_{sing}" should be "μ_{sing}".
- p. 155, middle: "$\{\{X_1 \le a\} \cap \{X_2 \le b\} : a, b \in \mathbf{R}\}$" should be "$\sigma(\{\{X_1 \le a\} \cap \{X_2 \le b\} : a, b \in \mathbf{R}\})$".
- p. 171, line 11: "$\mathbf{E}\left(\lim_{M\to\infty} U_M^{(\alpha,\beta)} = \infty\right) = \infty$" should be "$\mathbf{E}\left(\lim_{M\to\infty} U_M^{(\alpha,\beta)}\right) = \infty$".
- p. 178, line −6: "$\{X_{t_1}, \ldots, X_{t_k}) \in H\}$" should be "$\{(X_{t_1}, \ldots, X_{t_k}) \in H\}$".
- p. 192, Exercise 15.6.8: replace "$Z_t = \exp[-(a+\frac{1}{2}b^2)t+X_t]$" with "$Z_t = \exp[-2aX_t/b^2]$". (In fact, the first Z_t is also a martingale, but it is less useful than the second version.)
- p. 202, lines 14–15: to avoid subtleties about equivalent repeating decimals, perhaps replace "$c_i = 4$ if $d_i = 5$, while $c_i = 5$ if $d_i \ne 5$" with "$c_i = 2$ if $d_i \ge 5$, while $c_i = 7$ if $d_i < 5$". (This is not strictly necessary, but it makes the argument a bit cleaner.)
- p. 204, Exercise A.3.8: change "$\sum_{i=1}^{\infty}$" to "$\sum_{i=2}^{\infty}$" (twice), and "$\int_1^{\infty}$" to "$\int_2^{\infty}$".
- p. 204, Exercise A.3.9: change "$\sum_{i=1}^{\infty}$" to "$\sum_{i=3}^{\infty}$" (twice).
- p. 207, second line of Exercise A.5.1: "equivalence class" should be "equivalence relation".

Errata to Second Printing, 2007 (corrected in Fourth Printing, 2010):

[With thanks to Orn Arnaldsson, Bent Jørgensen, Chris Mansley, Kohei Nagamachi, Patrick Rabau, Mohsen Soltanifar, Hermann Thorisson.]

- p. 19, Exercise 2.5.6, and also p. 22, proof of Lemma 2.6.2: replace "$A_1, A_2, \ldots \in \mathcal{J}$" by "$A_1, A_2, \ldots$ are finite unions of elements of $\mathcal{J}$".
- p. 23, Exercise 2.6.4: "$\mathbf{P}(\emptyset) = 1$" should be "$\mathbf{P}(\Omega) = 1$".
- p. 23, Exercise 2.7.3, part (b): interchange "semialgebra" and "algebra". (Also, for stylistic improvement, swap parts (a) and (b).)
- p. 30, last line of proof of Proposition 3.1.5: "$\{X \le x\}$" should be "$\{Z \le x\}$".
- p. 33, line 7: second "$\mathbf{P}(X \in T)$" should be "$\mathbf{P}(Y \in S)$".
- p. 39, Exercise 3.6.8: exercise is correct, but special cases like "$d \le b + c - a$" and "$d > b + c - a$" should be modified.
- p. 40, Exercise 3.6.14: insert "independent" before "non-negative".
- p. 74, the proof of Lemma 7.1.2 is sloppily written and should be replaced by:

  Since F is right-continuous, we have that $\inf\{x; F(x) \ge u\} = \min\{x; F(x) \ge$

$u\}$, i.e. the infimum is actually obtained. It follows that $\phi(u) \leq x$ if and only if $u \leq F(x)$. Hence, since $0 \leq F(x) \leq 1$, we obtain that $\mathbf{P}(\phi(U) \leq x) = \mathbf{P}(U \leq F(x)) = F(x)$.

- pp. 113–114: "Example 9.5.9" should be "Exercise 9.5.9", and similarly for 9.5.11 and 9.5.12.
- p. 114, Exercise 9.5.12: "characteristic function" should be "moment generating function".
- p. 118 middle, the three lines following Figure 10.1.2: replace "g" by "f" (five times).
- p. 118, line -5: "$F(w) \geq b$" should be "$F(z) \geq b$".
- p. 121, Exercise 10.3.5: "four conditions" should be "five conditions".
- p. 126, statement of Lemma 11.1.2: "$\phi(t)$" should be "e^{itx}".
- p. 128, line -7: "π" should be removed from the equation.
- p. 131, line 9: "$\lim_n$" should be "$\lim_k$" (twice); "F_n" should be "F_{n_k}" (twice); and "μ_n" should be "μ_{n_k}".
- p. 136, lines 3–4: the sentence in brackets is somewhat misleading and should be revised or omited.
- p. 136, eqn (11.3.1): "$\mathbf{1}_{|Z_{nk} \geq \epsilon s_n}$" should be "$\mathbf{1}_{|Z_{nk}| \geq \epsilon s_n}$"
- p. 166, lines 6–7: "bets \$1 on tails, then if they win they bet \$2 on heads" should be "bets \$1 on heads, then if they win they bet \$2 on tails".
- p. 173, Exercise 14.4.1: should say "$\mathbf{P}(Z_i = 1) = \mathbf{P}(Z_i = 0) = 1/2$", and "$X_1 = 2Z_1 - 1$".
- p. 174, Execise 14.4.12(a), Hint: "$\mathbf{P}(\tau \geq 3m)$" should be "$\mathbf{P}(\tau > 3m)$".

Errata to First Printing, 2006 (corrected in Second Printing, 2007):

[With thanks to Joe Blitzstein, Saad Siddiqui, Emil Zeuthen.]

- p. 9, line 10 from bottom: "*all* intervals" should be "*all* subsets".
- p. 18, eqn. (2.5.2): "$P(B)$" should be "$\mathbf{P}(B)$".
- p. 19, the last sentence in the proof of corollary 2.5.4 is questionable (since we may have $D_n \notin \mathcal{J}$), and should be replaced by:

It then follows from (2.5.5) that

$$\mathbf{P}\left(\bigcup_n B_n\right) \;=\; \mathbf{P}\left(\bigcup_n D_n\right) \;=\; \mathbf{P}\left(\bigcup_n \bigcup_{i=1}^{k_n} J_{ni}\right) \;=\; \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni})\,.$$

On the other hand,

$$B_n \;=\; \bigcup_{m\le n} \bigcup_{i=1}^{k_m} (J_{mi} \cap B_n)$$

and the union is disjoint, with $J_{ni} \subseteq B_n$, so

$$\mathbf{P}(B_n) \;=\; \sum_{m\le n} \sum_{i=1}^{k_m} \mathbf{P}(J_{mi} \cap B_n) \;\ge\; \sum_{i=1}^{k_n} \mathbf{P}(J_{ni} \cap B_n) \;=\; \sum_{i=1}^{k_n} \mathbf{P}(J_{ni})\,,$$

and hence

$$\sum_n \mathbf{P}(B_n) \;\ge\; \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}) \;=\; \mathbf{P}\left(\bigcup_n B_n\right)\,.$$

- p. 20, eqn. (2.5.10): "$(\infty, x]$" should be "$(-\infty, x]$".
- p. 22, line 10 from bottom: "P_1" should be "$\mathbf{P}_1$", and "P_2" should be "$\mathbf{P}_2$".
- p. 151, first line of Section 13.1: "We being" should be "We begin".
- p. 162, last line: "$X_n = 5$" should be "$X_n = -5$".
- p. 205, Exercise A.4.5: "contraction" should be "contradiction".
- p. 206, line 4: "$g(x)/h(x)$" should be "$|g(x)/h(x)|$".
- p. 206, line 7: "limsup" should be "lim".

ERRATA FOR THE ON-LINE SOLUTIONS FILE:

- Exercise 2.7.22(a): add a description of Ω, and extend $\mathcal{F}$ to allow for separate subsets A and B, not just a single subset A.
- Exercise 9.5.14(a): answer should be 1, not ∞.