

**“Struck by Lightning” Supplementary Materials**

**Reminder About Solution to Gambler’s Ruin Problem.**

Recall that we were considering the following game (to be referred to as the original game). A starts with \( a \) pennies, and B starts with \( 8 - a \) pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then B gives one penny to A. If it comes up 3, 4, 5, or 6, then A gives one penny to B. This is repeated until either A or B wins all the pennies. That person is the “winner”. Recall that we wrote \( s(a) \) for the chance that A wins this game, starting with \( a \) pennies.

What follows is a very brief “reminder” of how we derived a formula for \( s(a) \). Learn it well; we will come back to these issues soon!

**Step #1.** Obviously \( s(0) = 0 \) and \( s(8) = 1 \).

**Step #2.** By considering what happens on the first bet, we see that

\[
s(a) = \frac{1}{3} s(a + 1) + \frac{2}{3} s(a - 1),
\]

for \( a = 1, 2, 3, 4, 5, 6, 7 \).

**Step #3.** Since \( s(a) = \frac{1}{3} s(a) + \frac{2}{3} s(a) \), this last formula can be re-written as

\[
s(a + 1) - s(a) = 2 [s(a) - s(a - 1)].
\]

**Step #4.** Hence, setting \( x = s(1) \), we see that

\[
s(1) - s(0) = x, \quad s(2) - s(1) = 2x, \quad s(3) - s(2) = 4x, \quad \text{etc.}
\]

and in general that

\[
s(a + 1) - s(a) = 2^a x
\]

for \( a = 0, 1, 2, 3, 4, 5, 6, 7 \).

**Step #5.** It follows that, for \( a = 0, 1, 2, 3, 4, 5, 6, 7, 8 \),

\[
s(a) = s(a) - s(0) = (s(a) - s(a - 1)) + (s(a - 1) - s(a - 2)) + \ldots + (s(1) - s(0))
\]

\[
= (2^{a-1} + 2^{a-2} + \ldots + 2 + 1) x = (2^a - 1) x.
\]

**Step #6.** Since \( s(8) = 1 \), it follows that \( x = \frac{1}{2^8 - 1} \), whence

\[
s(a) = \frac{2^a - 1}{2^8 - 1} = \frac{2^a - 1}{255}.
\]

So, for example, \( s(6) = (2^6 - 1)/255 = 63/255 = 0.2470588 \approx 24.7\% \).